Random Numbers on GPUs

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Introduction

- Artificial Intelligence needs Randomisation
- Implementing randomisation is hard.
- GPU no native support for bit level operations, long integers etc.
- Widespread fear of GPU implementation of random numbers.
- Demonstrate GPU can generate billions of random numbers.
- 400+ speedup v single precision Park-Miller

Need for Random Numbers

- Many Computational Intelligence techniques need cheap randomisation
  - Evolutionary computation: selection and mutation
  - Simulated Annealing
  - Artificial Neural Networks: random initial connection weights
  - Particle Swarm Optimisation
  - Monte Carlo methods, e.g. finance, option pricing

Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.

John von Neumann

- History of pseudo random numbers (PRNG) is littered with poor implementations. IBM's randu described by Knuth as "really horrible".
- Still true: bug in SUN's random.c
- Care needed when choosing random method Knuth

Park and Miller

- Park-Miller big study of linear congruent PRNGs. Fast.
- Suggest "minimal standard" PRNG.
- Uniform integer in 1 to 2^{31}
- Mandatory PRNG for proposed internet error correction standard.
- Requires 46 bits (Mersenne Twister ≈20k).
- 46 bits typically implemented using long int or double precision. Not available on GPU.

Park-Miller

```
intrnd (int& prng) {                    // 1<=prng<m
  int const a    = 16807;              //ie 7**5
  int const m    = 2147483647; //ie 2**31-1
        prng = (long(prng * a))%m;
}
```

- Next “random” number produced by multiplying current by 7^5 then reducing to range 1 to 2^{31} -2 using modulus `%m`
- Multiplication produces 46 bit result
- All calculations use integers

GPU Park-Miller

- C++ implementation under RapidMind
- GPU float only single precision
- Use Value4f (vector of 4 floats) to store and pass random numbers.
- 31 bits of Park-Miller broken into 4 bytes. Each byte stored as float. So no rounding problems.
- Value4f native GPU data type.

exactmul $7^5 \times$ Value4f → Value5f

```
exactmul(float f, float in[4], float out[5]) {
  out[0]=0;
  for(int i=0;i<4;i++) {
    const float t=in[i]*f;
    out[i]  += t;      //Max value 16807+16807×255
    out[i+1] = floor(out[i]/256);
    out[i]     =    int(out[i])%256;
  }
}
```

By performing multiplication a byte at a time calculation can be done with float

Parallel RapidMind exactmul

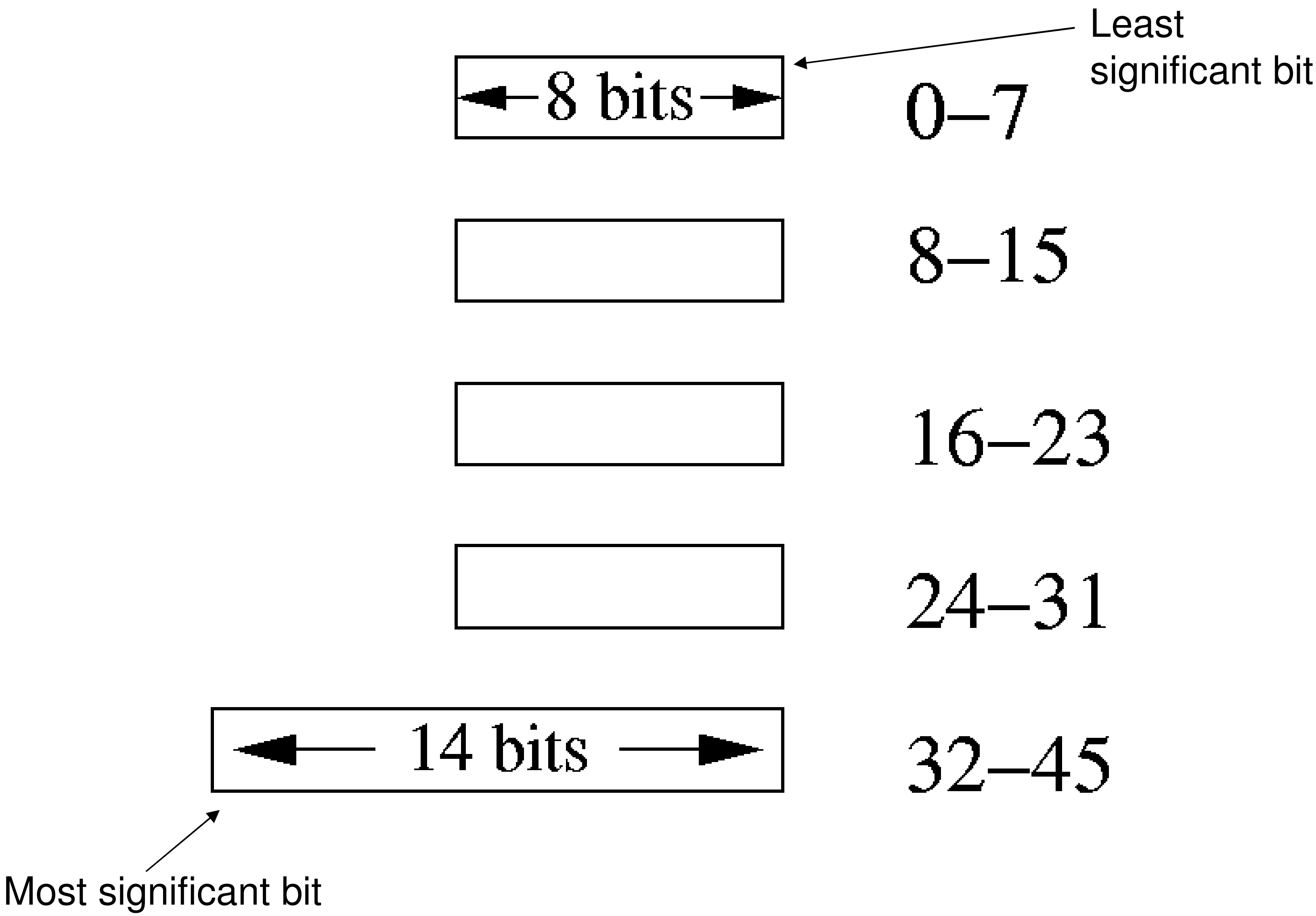
$7^5 \times$ Value4f $\rightarrow$ Value5f

```
inline void exactmul(const Value1f f, const Value4f in,
Value<5,float>& out) {
  //RM_DEBUG_ASSERT(f<= Value1f(16807));
  out[0]=0;
  for(int i=0;i<4;i++) {
    out[i]   += round(in[i]*f);
    out[i+1] = floor(out[i]/Value1f(256));
    out[i]     = round(Value1i(out[i])%256);
  }
}
```

multiplication a byte at a time so can be done with float.

round to ensure exact integer values.

Value5f used to represent 46 bits



$prng = (prng \times 7^5) \% 2147483647$

- After exactmul need to reduce modulus 2147483647 but 2^{31}-1 can not be represented exactly by float.
- Replace % by finding largest exact multiple of 2^{31}-1 which does not exceed $prng \times 7^5$ then subtract it from $prng \times 7^5$.
  - Avoids long division
- This gives the next Park-Miller pseudo random number.

Finding largest multiple of 2^{31}-1 not exceeding prng×7^5

- Find (approx) (prng×7^5)/(2^{31}-1)
- Refine approximation
- Multiply exact divisor by 2^{31}-1
- Obtain next PRNG by subtracting exact multiple of 2^{31}-1 from prng×7^5.
- Multiply and subtraction can be done exactly (using trick of spliting long integer into 8-bit bytes and storing these in floats).

Finding Largest Divisor 1

```
Value1i approxdiv  = floor(prng*a/m);
Value1i comp = -1;            // loop at least once
FOR(nul,comp<0,nul) {
    exactmul(Value1f(approxdiv),M,multiM);
    comp=comp5(out,multiM); //nb out=a*Prng
    approxdiv--;
}ENDFOR
```

$a = 7^5$ $M = 2^{31}-1$

- For loop used to decrement approxdiv until multiM=approxdiv$\times(2^{31}-1) \le$ prng$\times 7^5$
- Mostly only loop only used 1 or 2 times.

Finding Largest Divisor 2

```
exactsub5(out,multiM,Prng);   // prng = out-multiM
FOR(nul,comp4(Prng,M)>=0,nul) {
   exactsub4(Prng,M,Prng);    // prng=prng-m;
}ENDFOR
```

a= 7^5 M = 2^{31}-1

- In case approxdiv was too low the FOR loop is used to reduce the new PRNG by repeatedly subtracting 2^{31}-1 until it is below 2^{31}-1.

Comp4 using RapidMind

```
inline Value1i comp4(const Value4f a, const Value4f b) {
  return cond(a[3]>b[3],Value1i(+1),
        cond(a[3]<b[3],Value1i(-1),
        cond(a[2]>b[2],Value1i(+1),
        cond(a[2]<b[2],Value1i(-1),
        cond(a[1]>b[1],Value1i(+1),
        cond(a[1]<b[1],Value1i(-1),
        cond(a[0]>b[0],Value1i(+1),
        cond(a[0]<b[0],Value1i(-1),Value1i(0)))))))));
}
```

Use GPU cond to compare most significant parts of a and b first

Exactsub5 using RapidMind

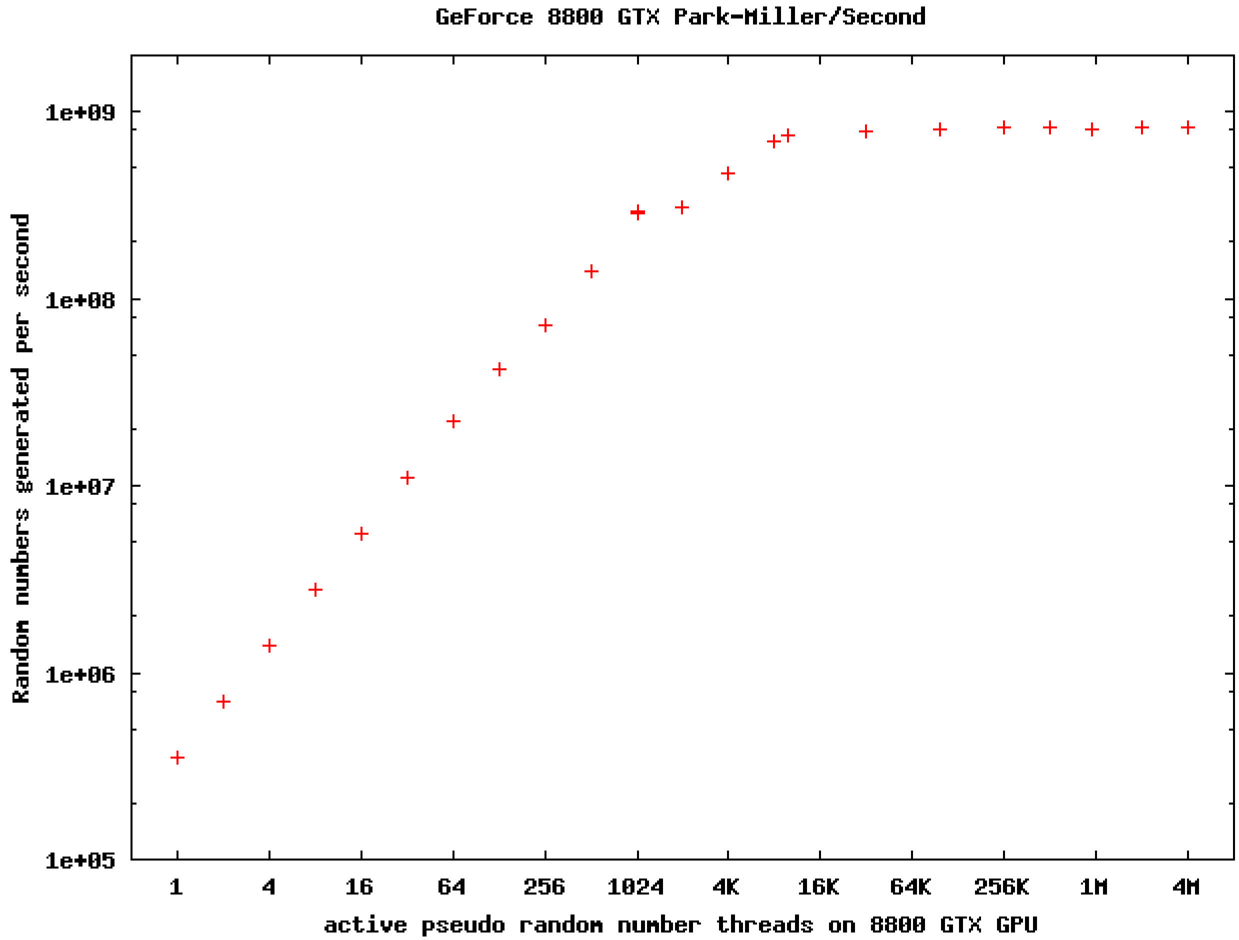
- Operate on local copies of inputs to avoid side effects on calling code.
- Requires a≥b and a-b fits in 4 bytes
- Subtract B[i] from A[i]. Use round to force integer
- If A[i]<B[i] "borrow" 256 from B[i+1].
- No negative values

```
//nb a>=b
for(int i=0;i<4;i++) {
   B[i+1] = cond(A[i]<B[i],round(B[i+1]+Value1f(1)),B[i+1]);
   A[i]   = cond(A[i]<B[i],round(A[i]+Value1f(256)),A[i]   );
   out[i] = round(A[i]-B[i]);
 }
//A[5]==B[5]
```

Validation

- RapidMind GPU and two PC version of Park-Miller were each validated by generating at least the first 100 million numbers in the Park-Miller sequence and comparing with results in Park and Miller's paper and www.

Performance v threads



In test environment, with ≥ 8192 threads the 128 stream processors give peak performance. I.e. ≥16 active threads per SP. Or ≥512 threads per G80 8SP block.

Performance

- nVidia GeForce 8800 GTX (128 SP)
- 833 10^6 random numbers/second
- 44 times double precision CPU (2.40Ghz)
- More than 400 times single precision CPU
- Estimate 90 GFlops (17% max 518.4 nVidia claim)

Discussion

- 90GFlops too high?
- Test harness semi-realistic.
- GPU application, PRNG just a small part, but avoids communication with CPU.
- Main bottle neck is access to GPU's main memory.
- PRNG faster if use on-chip memory but application may want this for other reasons.
- Importance of many threads (min 512).

Conclusions

- Cheap randomisation widely needed but often poorly implemented.
- Fear of PRNG on GPU (said GPU cant do)
- Park-Miller fast but needs more than float
- GPU implementation meets Park and Miller's minimum recommendation.
- RapidMind C++ Code available via ftp.
- Up to 833 million pseudo random numbers per second.

END

Questions

- Code via ftp
  - http://www.cs.ucl.ac.uk/staff/W.Langdon/ftp/gp-code/random-numbers/gpu_park-miller.tar.gz
- gpgpu.org GPgpgpu.com rapidmind.com