

## 2010: Compilers

### Syntax Analysis – Context Free Grammars

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UCL/CS

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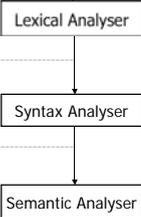
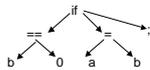
### WHERE WE ARE

Source code (character stream)

if (b==0) a=b;

Tokens stream  
IF LPAREN ID(b) EQ NUM(0)  
RPAREN ID(a) BECOMES ID(b) SEMI

Abstract Syntax Tree (AST)




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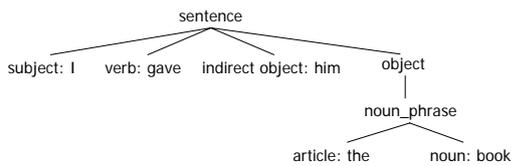
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### SECOND STEP: SYNTAX ANALYSIS

- Goal: determine if the input token stream satisfies the syntax of the program

"I gave him the book"




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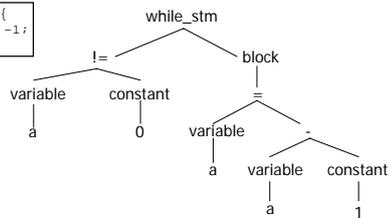
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### SECOND STEP: SYNTAX ANALYSIS

- Goal: determine if the input token stream satisfies the syntax of the program

```
while (a!=0) {  
    a = a -1;  
}
```



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### SECOND STEP: SYNTAX ANALYSIS

- What we need for syntax analysis:
  - An expressive way to describe the syntax
    - ... why not regular expressions?
  - An acceptor mechanism that determines if the input token stream satisfies the syntax description

... why not DFA?

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### SECOND STEP: SYNTAX ANALYSIS

- Example: nested constructs  
{} {} {}{}{} ...

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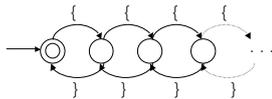
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## SECOND STEP: SYNTAX ANALYSIS

- Example: nested constructs

{ } { } { } { } { } { } ...

- RE are not powerful enough to express the syntax of a programming language



We need unbounded counting!

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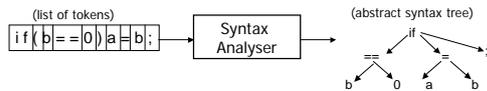
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## PHASE 2 - OUTLINE

- Syntax Analyser



- Context-Free Grammars (CGF)
- Acceptors: LL(k), LR(K), SLR, LALR
- Parser Generator (CUP)

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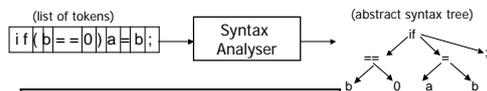
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## PHASE 2 - OUTLINE

- Syntax Analyser



- Context-Free Grammars (CGF)
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### CONTEXT FREE GRAMMARS (CFG)

- Language L = set of strings  
... *programs*
- String = finite sequence of symbols  
... *lexical tokens*
- Symbols = taken from finite alphabet A  
... *set of token types*

A Context-Free Grammar CFG describes a language L(CFG)

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### CONTEXT FREE GRAMMARS (CFG)

- Context-Free Grammar (CFG):
  - Terminal symbols = token or  $\epsilon$
  - Non-terminal symbols = syntactic variables
  - Start symbol = special non-terminal
  - Productions of the form LHS  $\rightarrow$  RHS
    - LHS: a single non-terminal
    - RHS: both terminals and non-terminals
    - $\rightarrow$ : specify how non-terminals can be expanded
- Language L(G) generated by a CFG G = set of strings of *terminals* derived from the start symbol by repeatedly applying the productions

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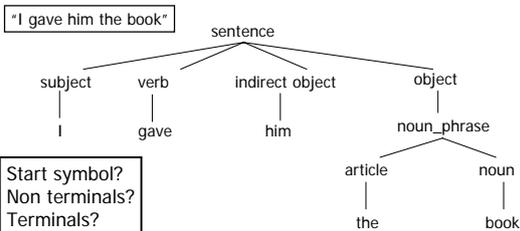
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### CONTEXT FREE GRAMMARS (CFG)



Start symbol?  
Non terminals?  
Terminals?  
Productions?

"I gave him books"

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### CONTEXT FREE GRAMMARS (CFG)

- Example: language of balanced parenthesis

Terminals	{ }
Non-terminals	S
Start Symbol	S
Productions	$S \rightarrow \{S\}S$ $S \rightarrow \epsilon$

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### CONTEXT FREE GRAMMARS (CFG)

- A string is in  $L(G)$  if it exists a derivation of that string
- Example: is  $\{\{\}\}\{\}$  in  $L(G)$ ?

$$S \rightarrow \{S\}S$$

$$S \rightarrow \epsilon$$

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### CONTEXT FREE GRAMMARS (CFG)

- A string is in  $L(G)$  if it exists a derivation of that string
- Example: is  $\{\{\}\}\{\}$  in  $L(G)$ ?

$$S \rightarrow \{S\}S$$

$$S \rightarrow \epsilon$$

$$\begin{aligned}
 S &\rightarrow \{S\}S \rightarrow \{\{S\}S\}S \rightarrow \{\{\{S\}S\}S\}S \rightarrow \{\{\{\{S\}S\}S\}S\}S \rightarrow \\
 \{\{\}\}\{S\} &\rightarrow \{\{\}\}\{\{S\}S\} \rightarrow \{\{\}\}\{\{\}\}\{S\} \rightarrow \{\{\}\}\{\{\}\}\{\epsilon\} \rightarrow \{\{\}\}\{\{\}\}\{\}
 \end{aligned}$$

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### CONTEXT FREE GRAMMARS (CFG)

- Example: simple calculator

Terminals	+ * id ( )
Non-terminals	E
Start Symbol	E
Productions	$E \rightarrow E + E$ $E \rightarrow E * E$ $E \rightarrow id$ $E \rightarrow ( E )$

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### CONTEXT FREE GRAMMARS (CFG)

- Is  $id + id * id$  in  $L(G)$ ?

1.  $E \rightarrow E + E$
2.  $E \rightarrow E * E$
3.  $E \rightarrow id$
4.  $E \rightarrow ( E )$

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### CONTEXT FREE GRAMMARS (CFG)

- Is  $id + id * id$  in  $L(G)$ ?

1.  $E \rightarrow E + E$
2.  $E \rightarrow E * E$
3.  $E \rightarrow id$
4.  $E \rightarrow ( E )$

$$\begin{aligned}
 E &\xrightarrow{1.} E + E \xrightarrow{3.} id + E \xrightarrow{2.} id + E * E \xrightarrow{3.} id + id * E \\
 &\xrightarrow{3.} id + id * id
 \end{aligned}$$

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### CONSTRUCTING A DERIVATION

- Start from start symbol S
- Use productions to derive a sequence of tokens from the start symbol
- For arbitrary strings  $\alpha$ ,  $\beta$ , and  $\gamma$ , and for a production:

$$A \rightarrow \beta,$$

a single step of derivation is:

$$\alpha A \gamma \rightarrow \alpha \beta \gamma$$

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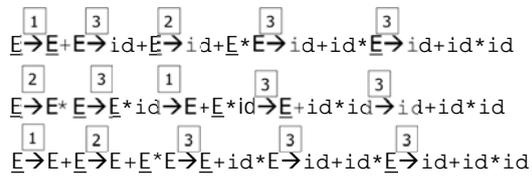
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### CONSTRUCTING A DERIVATION

- Derivations:
  - Left-most=the left-most non-terminal symbols is always the one expanded
  - Right-most=the right-most non-terminal symbol is always the one expanded




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### DERIVATIONS AND PARSE TREES

- Parse Tree = graphical (tree) representation of a derivation
  - Leaves = terminals
  - Intermediate nodes = non-terminals
  - Root = start symbol

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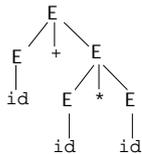
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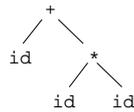
### DERIVATIONS AND PARSE TREES

$E \rightarrow E + E \rightarrow E + E * E \rightarrow id + E * E \rightarrow id + E * id$   
 $\rightarrow id + id * id$

Parse Tree  
(concrete syntax tree)



Abstract Syntax Tree



ASTs contain only terminals

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### EXERCISE

- Given the grammar

$S \rightarrow ( L ) \mid a$   
 $L \rightarrow L , S \mid S$

- Construct a left-most derivation for (a, (a,a))
- Construct a right-most derivation for (a, (a,a))
- Build the parse tree and the abstract syntax tree for the above derivations

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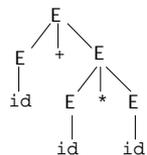
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### DERIVATIONS AND PARSE TREES



$E \rightarrow E + E \rightarrow E + E * E \rightarrow id + E * E \rightarrow id + E * id$   
 $\rightarrow id + id * id$

$E \rightarrow E + E \rightarrow id + E \rightarrow id + E * E \rightarrow id + id * E$   
 $\rightarrow id + id * id$

- Note: parse trees contain no information about the order of the derivation steps
- Different derivations may have the same parse tree

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**RECAP ...**

- Context-Free Grammar (CFG) describes the language of syntactically correct program
  - Correctness: it exists a derivation that produces the input token list
    - Left-most vs. right-most derivation
  - Parse-tree: graphical representation of a derivation
    - Concrete vs. abstract syntax tree

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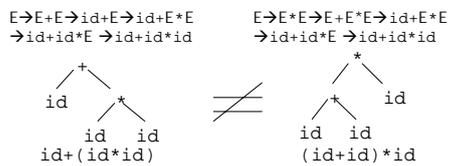
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**DERIVATIONS AND PARSE TREES**

- Ambiguous grammar = there exist different left-most (or right-most) derivations for the same string
  - These derivations have different parse trees (thus different meaning)




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**ELIMINATING AMBIGUITY**

- Ambiguous grammars should be avoided
- Unambiguous CFGs specify how to convert a token stream into a **unique** parse tree
- Eliminating ambiguity
  - Heuristics
    - Adding non-terminals to enforce precedence
    - Allowing only left or right recursion (for associativity)

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**ELIMINATING AMBIGUITY**

- Ambiguous grammars should be avoided
- Unambiguous CFGs specify how to convert a token stream into a **unique** parse tree
- Eliminating ambiguity
  - Heuristics
    - \* has higher precedence than +:
      - a+b\*c means a+(b\*c)
    - Each operator associates to the left:
      - a-b-c means (a-b)-c

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**ELIMINATING AMBIGUITY**

G1:  $E \rightarrow E+E$       G2:  $E \rightarrow E+T \mid T$   
 $\mid E*T$                      $T \rightarrow T*F \mid F$   
 $\mid (E)$                      $F \rightarrow (E) \mid id$   
 $\mid id$

id+id\*id

$\underline{E} \rightarrow \underline{E}+T \rightarrow \underline{I}+T \rightarrow \underline{E}+T \rightarrow id+\underline{I} \rightarrow id+\underline{I}*F \rightarrow$   
 $id+\underline{F}*F \rightarrow id+id*\underline{F} \rightarrow id+id*id$

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**EXAMPLE**

- Grammar for if-then-else
  - $S \rightarrow if (E) S$
  - $S \rightarrow if (E) S else S$
  - $S \rightarrow \dots$
- Is this grammar ok?

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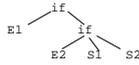
**EXAMPLE**

$S \rightarrow \text{if}(E) S$   
 $S \rightarrow \text{if}(E) S \text{ else } S$

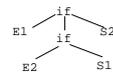
- How to parse:

$\text{if}(E_1) \text{if}(E_2) S_1 \text{ else } S_2$

$S \rightarrow \text{if}(E) S \rightarrow \text{if}(E) \text{if}(E) S \text{ else } S$



$S \rightarrow \text{if}(E) S \text{ else } S \rightarrow \text{if}(E) \text{if}(E) S \text{ else } S$




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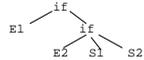
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**EXAMPLE**

- Closest-if rule

$\text{statement} \rightarrow \text{matched} \mid \text{unmatched}$   
 $\text{matched} \rightarrow \text{if}(E) \text{matched} \text{ else } \text{matched} \mid S$   
 $\text{unmatched} \rightarrow \text{if}(E) \text{statement} \mid$   
 $\text{if}(E) \text{matched} \text{ else } \text{unmatched}$

$\text{statement} \rightarrow \text{unmatched} \rightarrow \text{if}(E) \text{statement} \rightarrow$   
 $\text{if}(E) \text{matched} \rightarrow \text{if}(E) \text{if}(E) \text{matched} \text{ else } \text{matched} \rightarrow$   
 $\text{if}(E) \text{if}(E) S \text{ else } \text{matched} \rightarrow$   
 $\text{if}(E) \text{if}(E) S \text{ else } S$




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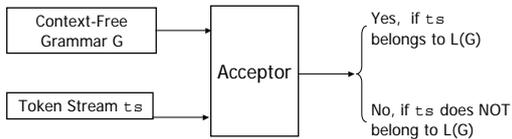
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**WHAT'S NEXT?**

- Acceptors for context-free grammars



- Syntax Analysers (parsers) = CFG acceptors which also output the corresponding derivation when the token stream is accepted

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