A Generalisable Measure of Self-Organisation and Emergence

W.A. Wright[1], R. E. Smith[2], M. Danek[2], and P. Greenway[1]

[1] British Aerospace Systems ATC Sowerby, Bristol, UK,
{andy.wright,phil.greenway}@src.bae.co.uk
[2] The Intelligent Computing Systems Centre, Bristol, UK,
{robert.smith,martin.danek}@uwe.ac.uk

Abstract. In adaptive systems that involve large numbers of entities, emergent, global behaviours that arise from localised interactions are a critical concept. Understanding and shaping emergence may be essential to such systems' success. To aid in this understanding, this paper introduces a measure gleaned from non-linear systems theory. The paper discusses how this measure can be used in reinforcing self organising behaviours in adaptive systems. Further, it is shown that the measure can be successfully employed as feedback to a system employing evolutionary computation (EC) and using this to design in desired self organising behaviours in an approximation to a biological plausible collective system.

1 Introduction

As the complexity of computing systems has increased, the presence of undesirable behaviours that are a result of unforseen non-linear interactions with the different components of these systems has grown. These behaviours can have catastrophic consequences, as in the sudden collapse of the eastern seaboard telephone system in the USA [4]. It has also been argued that similar behaviour is also apparent in other complex systems, such as: crowds of people, traffic systems (i.e., the stop start behaviour in motorway traffic jams). Biological systems also appear to demonstrate such behaviour. Such as the sudden dispersion a schools of fish in the presence of a predator [3]. In this last case what may be termed as an emergent property of the system appears to provide a behaviour that is advantageous to the group as a whole. This natural interest in the behaviour of social animals coupled with the possibility of utilising these behaviours to design complex systems (including communications, robotic [5], and artificial intelligence systems) has prompted the study of the self-organisation in collective biological system such as flock and herds of animals.

Although a substantial element of this research has successfully concentrated on building mathematical [10, 7] and rule-based [6] models that simulate the dynamics of these systems, the control of these models is difficult. To utilise the inherent non-linear interactions that are present in many complex systems, one would ideally like to be able to produce a desired behaviour by simply choosing the appropriate parameters values for a system. This has not been possible. An alternative is to adjust the parameters (via some optimisation routine) such that the system emulates a desired behaviour. However, to do this, as has recently been pointed out by Zaera et al [12], requires some qualitative measure of the behaviours of the system. In the past, Lyapunov exponents have been used to measure the behaviour of flocking systems [9]. Unfortunately, this approach is limited, since the Lyapunov exponent only measures the stability of a system's dynamics and does not consider the level of coherence (or structure) in the behaviours exhibited by a system. This paper presents an alternative measure based on Takens' method of delays [8] and the embedding approach of Broomhead and King [1], to derive an entropic measure that describes the complexity of the system. This measure is then used as direct feedback to evolutionary computation (EC) systems that evolve swarm-like behaviours in a simple, Hamiltonian-based flocking model. The implications of this work for other complex systems is discussed in future work.

2 Continuous time model of flocking

This section briefly describes a simple continuous time model of a flock of "particles" that is used to illustrate the utility of complexity measure derived in this paper and its subsequent use to optimise the behaviour of such a system. In this model it is assumed the a self organising set of N particles can be described by a number of local state-space variables. For the simple flocking model these are the set of positions and velocities for all the flock members. The dynamics of these agents is controlled by the local interactions between each agent and its neighbours that control the spatial and velocity coherence of the system.

The spatial coherence between the particles is modelled using the Hamiltonian

$$\mathcal{H} = 1/2m \sum_{i}^{N} |\dot{\boldsymbol{x}}_i|^2 - V_0 \sum_{i}^{N} \sum_{j \neq i}^{N} V(r_{ij}), \tag{1}$$

where m is the particle mass $\boldsymbol{x}_i$ is the positional vector for the particle i, r_{ij} is the Euclidian distance between the i^{th} and j^{th} particle and $V(r_{ij})$ is a hydrogen potential with depth V_0. The velocity coherence is modelled using a Stokes viscous term

$$\mathcal{R} = 1/2\mu \sum_{i}^{N} \sum_{j:r_{ij} \leq r_1}^{N} |\dot{\boldsymbol{x}}_i - \dot{\boldsymbol{x}}_j|^2, \tag{2}$$

where μ is the viscosity coefficient and r_1 is a effective range of this term. Solving for the equations of motion gives:

$$\ddot{x}_i = \gamma \sum_{j \neq i} \frac{\partial V(r_{ij})}{\partial x_i} + \delta \sum_{i}^{N} \sum_{j:r_{ij} \leq r_1}^{N} (\dot{x}_i - \dot{x}_j), \tag{3}$$

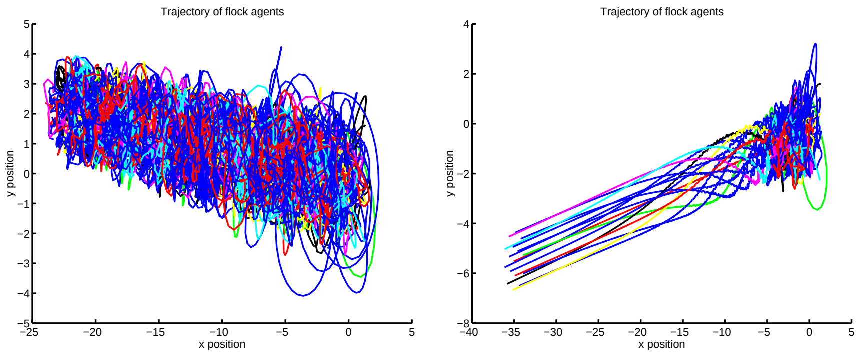
where the couplings between the spatial and damping terms that control the dynamics have been rewritten as:

$$\gamma = \frac{V_0}{r_0^2 m}, \tag{4}$$

and

$$\delta = \frac{\mu}{m}, \tag{5}$$

where r_0 is derived from the hydrogen potential and controls the optimal spacing of the particles. Solving this continuous time model gives rise to a number of distinctly different behaviours as the two control parameters γ and δ are varied. Two extreme behaviours (an uncoordinated "swarm" like behaviour and a coordinated "crystal" like behaviour) are shown in Figure 1



(a) Swarm

(b) Crystal

Fig. 1. Trajectories of 17 particles illustrates two different behaviours. Note that the particles start at the right of the plot, and proceed to the left.

3 Entropic Measure

The measure derived in this paper is based on the idea that although a complex system may have many degrees of freedom, the self-organised or emergent behaviour of that system can be modelled using noticeably fewer degrees of freedom. This is illustrated in the "crystalline" behaviour of the above flock model. Here the particles (which have in total 17×4 degrees of freedom) move as a single body with 4 degrees of freedom. In non-linear dynamical systems this self-organising behaviour is likened to the system moving on some attractor which has a much smaller dimension m compared to the dimension of the whole system.

Takens [8] has shown that it is possible to determine dimension of the attractor that generates the time-series from successive observations of the time-series. An approximation to this is approach is provided by Broomhead and King [1] where an integer bound on the dimension of the attractor can be estimated from the singular values σ of the embedding matrix. For the 2D flocking system described above this matrix is constructed from M time delay samples of the time-series generated by tracking each particle over time to give:

$$Z = \begin{pmatrix} x_1^1, & y_1^1, & \dot{x}_1^1, & \dot{y}_1^1, & \ldots, & x_1^N, & y_1^N, & \dot{x}_1^N, & \dot{y}_1^N \\ & & & & \vdots & & & & \\ x_M^1, & y_M^1, & \dot{x}_M^1, & \dot{y}_M^1, & \ldots, & x_M^N, & y_M^N, & \dot{x}_M^N, & \dot{y}_M^N. \end{pmatrix} \quad (6)$$

Here x_j^i is the j^{th} time sample of the x coordinate of the i^{th} agent and N is the total number of agents in the flock. To ensure that any correlations in the data are removed, the mean position and velocity were subtracted from the positional and velocity data during the analysis giving a total of $4 \times (N-1)$ degrees of freedom. These values can then be used to construct an entropy

$$S = \sum_i^M \sigma_i' \log_2 \sigma_i', \quad (7)$$

where σ' is the normalised singular value. Since entropy can be seen as a log count of the number of states in a system the effective number of degrees of freedom and so the complexity is given by the expression

$$\Omega = 2^{\sigma'}. \quad (8)$$

This approach provides a easy to compute and robust method to quantitatively estimate the complexity of the dynamics of the flocking system. Furthermore, has been shown [1] that the method is robust to additive noise since in general the noise only influences the value of the smaller singular values that provide only a small contribution to the entropy.

Using this measure it is possible to show that the flocking system described above radically changes behaviours for smooth changes in parameter values. Figure 2 shows a plot of Ω versus two dimensionless parameters ($\alpha = \frac{\mu^2 r_0^2}{m V_0}$) which measures the relative strength of the spatial potential and viscous term and ($\beta = r_1/r_0$,) which is the ratio of the effective range of these terms. From this it can be seen that there is an abrupt change (possibly a phase transition) between the "swarm" (the upper surface with large Ω) and the "crystal" behaviours (the lower surface with small Ω).

To summarise and clarify, we associate a system's ability to exhibit emergent behaviour with sudden transitions in the Ω measure, relative to smooth changes in system parameters. Emergent behaviours are those whose parameter values are associated with these abrupt transitions in Ω. Further, note that this measure is generalisable to any complex system comprised of many elements, in that

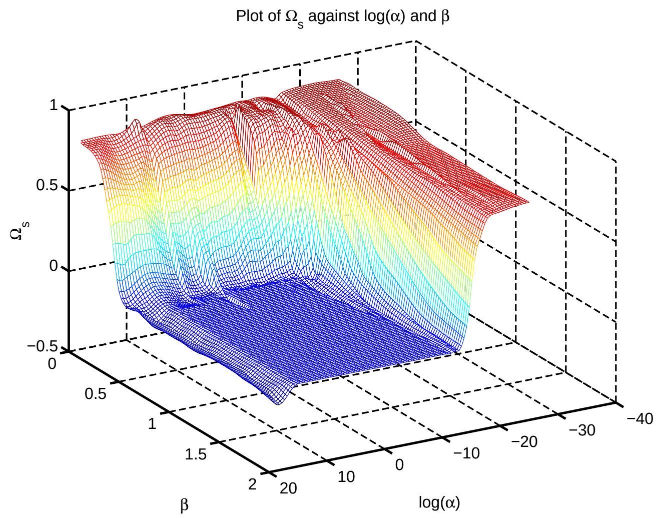


Fig. 2. Behaviour of our simple, illustrative system versus system parameters. This is a plot of Ω scaled by the total number of degrees of freedom versus two dimensionless parameters.

it only relies on the selection of a set of local state variables that can be to associated with each element. Its use of the method of delays allows its consideration of second or higher order effects, without the need to explicitly include second or higher-order derivatives as state variables for each element. Therefore, it is likely to have broad utility in the design and analysis of many types of complex systems, including communications, robotic, and artificial intelligence systems.

4 Using the Measure as Feedback for Adaptation

General-purpose adaptive and collective systems (e.g. collective mobile robotics, adaptive rule-based systems, etc.) each involve the ability to exploit the complex (non-linear) interactions that exist between the different elements that make up the collective. While it is possible, in a stable environment, that one could directly program such a system, explicitally allowing for the different interactions between all the agents, this is, by far, a much more difficult task to do if the system adapts to a changing environment. Here it is necessary it is to include some form of feedback mechanism, or use some form of online learning that will ensure the stability of the system.

We can illustrate how the measure discussed above can be used as feedback in an adaptive system to encourage self organisation, by using the measure as feedback to a genetic algorithm (GA) [2]. Initial experiments have demonstrated that it is possible to achieve behaviours in the simple flocking model that can be seen as mixtures of the two main behaviours (crystal and swarm). Figure 3 shows such a hybrid behaviour[1] that was generated using a an intermediate value of the scaled omega ($\Omega = 0.4$). Here the particles oscillate around each other. Other, more complex behaviours have been obtained using heterogenous systems of particles [11].

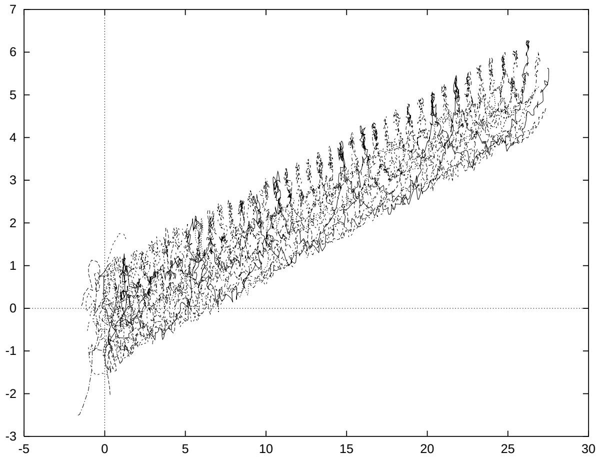
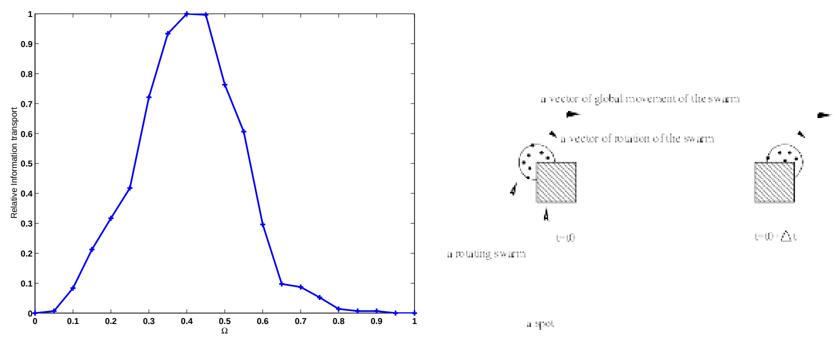


Fig. 3. Trajectories of 17 particles illustrating a hybrid behaviour.

More recent investigations have looked at the use of this measure for the online adaptation of a collections of interacting particles in a changing environment. In these experiments, the collective particle are placed in an environment with periodic boundary conditions (a toroid). This environment contains a small region where the particles are able to pick up some "food". Once an particle has this food, it local attraction to other particles is increased by a proportion v. When one particle contacts another, a proportion p of the food available to one particle is passed to the other. The amount of food that an individual particle holds decays exponentially with time, with a half-life λ. The aim of this experiment is to determine what type of behaviour maximises the propagation of food through the population of particles. Examination of this system has shown that the maximum propagation of the food throughout the population corresponds to $\Omega \sim 2.5$ for $\lambda = 0.5$ (see Figure 4a). Then, when this "target" Ω is used as a basis for fitness, a GA optimisation procedure finds other results for differing λ, v and p. Generally these results show a behaviour where the particles swarm initially to find the food and then orbit the food source, with sufficient oscillatory behaviour to ensure the propagation of the food to particles outside

[1] It is difficult to see the full dynamic behaviour from this still figure. However, animations of the dynamics will be presented with the full paper.

the food region[2] (see Figure 4b). Furthermore, as the proportion of food in the population drops, the strategy adopted by the particles is to increase the size of the swarm, thus increasing the cross section of the swarm, relative to the food source. Current work is using this measure to adjust a heterogenous group of particles, and looking at how adapting the behaviour of the individual particles can maximise the transport of food through the population.



(a) (b)

Fig. 4. (a) graph showing the Ω plotted against the transport of food in the population. (b) a schematic of the swarm behaviour around the food source.

5 Final Comments and Future Directions

We have introduced a means of quantifying the notion of self-organisation in multi-element systems, and illustrated how that measure can be used to obtain a desired emergent behaviour, via feedback to a GA. The measure, which is based on ideas borrowed from dynamical systems analysis can be calculated directly from data pertaining to the individual elements in a system. Theoretical studies of the embedding method that underpins this approach have shown that it is robust to levels of adaptive noise [1]. Consequently, although the utility of the measure has only been demonstrated on a model with very limited biological plausibility, its hope that it can be applied in a much more general fashion. We believe this measure (and ones like it) could be used as general-purpose feedback to evaluate and encourage desirable behaviours in a variety of complex systems, including artificial neural systems, evolutionary computation

[2] Again it is difficult to show the dynamic behaviour of this system. However, an animation of this result can be obtained at http://dual.felk.cvut.cz/ danek/research/mas.html

systems, and reinforcement learning systems. These applications are a focus of the author's current investigations.

References

1. D.S. Broomhead and G.P. King. Extracting quantitative dynamics from experimental data. *Physica D*, 20:217–236, 1998.
2. D. E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning.* Addison-Wesley, 1989.
3. S.A. Gueron, S. Levin, and D.I. Rubenstein. The dynamics of herds: from individuals to aggregations. *Journal of Theoretical Biology*, 182:85–98, 1996.
4. D. R. Kuhn. Sources of failure in the public switched telephone network. *Computer*, 3(4):31–36, 1997.
5. J. M. Maja. Behavior-based robotics as a tool for synthesis of artificial behavior and analysis of natural behavior. *Trends in Cognitive Science*, 2(3):82–87, 1998.
6. C. W. Reynolds. Flocks, herds, and schools: A distributed behavioral model. In *ACM SIGGRAPH '87 Conference Proceedings*, volume 21, pages 25–34. ACM Press, 1987.
7. N. Shimoyama, K. Sugawara, T. Mizuguchi, Y. Hayakawa, and M. Sano. Collective motion in systems of mobile elements. *Physical Review Leters*, 1996.
8. F. Takens. Detecting strange attractors in turbulence. In D.A. Rand and L.S. Yong, editors, *Lecture notes in mathematics*, page 366. Springer, Berlin, 1981.
9. J. Toner and Y. Tu. Flocks, herds, and schools: A quantitative theory of flocking. *Phys Rev E*, 58(4), 1998.
10. T. Vicsek, A. Czirol, E. Ben-Jacob, I. Cohen, and O. Shochet. Novel type of phase transition in system of self-driven particles. *Phy Rev E*, 75(6):1226–1229, 1995.
11. W. A. Wright, R. E. Smith, M. A. Danek, and P. Greenway. A measure of emergence in an adapting, multi-agent context. In *Supplmental Proceedings of the Sixth Intenational Conference on the Simulation of Adaptive Behaviour*, pages 20–27, Honolulu, 2000. ISAB.
12. N. Zaera, D. Cliff, and J. Bruten. Not evolving collective behaviours in synthetic fish. In *Proceedings of the Fourth International Conference on the Simulation of Adaptive Behaviour*, pages 635–644. MIT Press, 1996.