2011 INTRODUCTION TO GRAPHICS NOTES

ADDITIONAL NOTES AND EXERCISES

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LECTURE 6: SHAPES AND CURVES

CURVES

In this case a Quadric curve is defined by three points (two end-points and a control point) and Cubic curve is defined by four points (two end-points and two control points).

If you think about what a two-dimensional quadric curve might contain:

$$f(x,y) = ax^2y^2 + bx^2y + cxy^2 + dx^2 + exy + fy^2 + gx + hy + i$$

it should be obvious that 3 points, or 6 parameters are not enough to completely determine a quadric equation. A "Quad2D" curve is a simplification that allows simple editing.

MATH REVISION

Pick your favourite equation. For a circle an explicit form might be

$$y = \pm\sqrt{r^2 - x^2}$$

in general, explicit forms of equations are hard to find. Even if they can be found, they may require complex special cases to draw.
The implicit form of the circle equation is

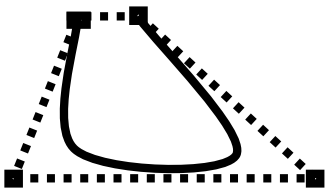
$$f(x,y) = x^2 + y^2 - r^2 = 0$$

which is even harder to draw. Implicit form are more general than explicit forms, but there is often no way to turn an implicit form into an explicit form.
Parametric forms, if they can be found, are easy to draw.

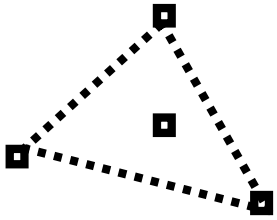
CUBIC

Aside … Bezier worked for Renault designing smooth shapes for cars.

The convex hull is the "outer" wrapper of the point set. With four points this is fairly clearly usually a quadrilateral



But with four points, we can also have a triangle as the convex hull:

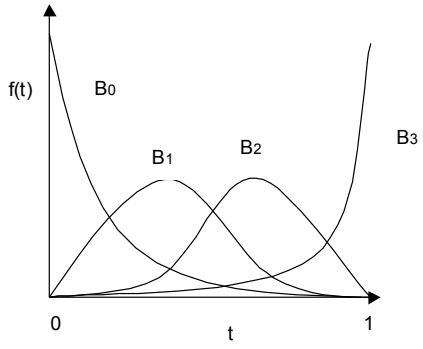


CONSTRUCTION OF BEZIER

Note that the split does not have to be 50-50 each time. It is possible to so the split in any proportion, as long as the same relative proportion is kept each time. This will, of course, bias the detail on the curve.

FORM OF THE EQUATION

The weighting functions look like the following:



The B is short for Bernstein polynomial.

EXERCISES

1. Suppose we are given the parametric curve defined by:

   $P(t) = (X(t),Y(t)) = (1 + t + t^2, t^3)$

   with t in the range 0 to 1. Find the Bezier control points for this curve.