Problems with ROC Convex Hulls
RN/01/16

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[Scott *et al.*, 1998] say the "Best" way of combining two (or more) classifiers is by taking the convex hull on the ROC[1] curve. I.e. a random linear combination of them both.

Also what do we mean by "Best"? If the goal is to ensure the largest possible number of positive cases are detected the union of the two classifiers should be used. (N.b. this assumes we are not concerned about the number of negative cases incorrectly classified as positive).

1 An example

Figure 1 shows an example of a deterministic classification problem. Given the features (inputs) the data lie in one of two classes, called positive and negative. The response of two classifiers X and Y are plotted in Figure 1 as our their union (X u Y) and intersection (X n Y).

The performance of the classifiers X and Y in terms of the fraction of positive examples correctly classified as positive (True Positives, TP) etc. are calculated in Table 1 and shown graphically in Figure 2. Also calculated are the performance of two classifier created by taking the union of X and Y and from the intersection of X and Y. **Note the performance of both of the combined classifies is superior to that indicated by the convex hull of X and Y.**

References

[Scott *et al.*, 1998] M. J. J. Scott, M. Niranjan, and R. W. Prager. Realisable classifiers: Improving operating performance on variable cost problems. In Paul H. Lewis and Mark S. Nixon, editors, *Proceedings of the Ninth British Machine Vision Conference*, volume 1, pages 304–315, University of Southampton, UK, 14-17 September 1998. `http://www.bmva.ac.uk/bmvc/1998/pdf/p082.pdf`.

[1] Receiver Operating Characteristics

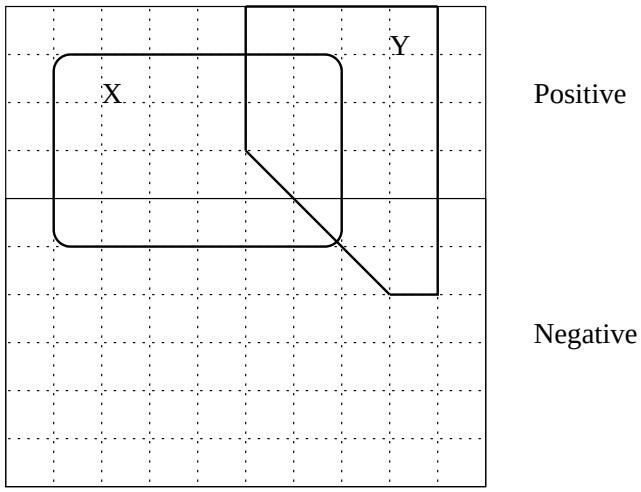

Figure 1: Data points above horizontal line are in the class, while those below it are not. The deterministic classifiers X and Y return true if the input lies within them and otherwise false.

Table 1: Classifier Accuracies

Classifier	True Positive	True Negative	False Positive	False Negative
X	18/40 = 0.45	54/60	6/60 = 0.1	22/40
Y	15.5/40 = 0.3875	56/60	4/60 = 0.066666	24.5/40
X u Y	28/40 = 0.7	50.5/60	9.5/60 = 0.158333	12/40
X n Y	5.5/40 = 0.1375	59.5/60	0.5/60 = 0.008333	34.5/40
1	40/40 = 1	0/60	60/60 = 1	0/40
0	0/40 = 0	60/60	0/60 = 0	40/40

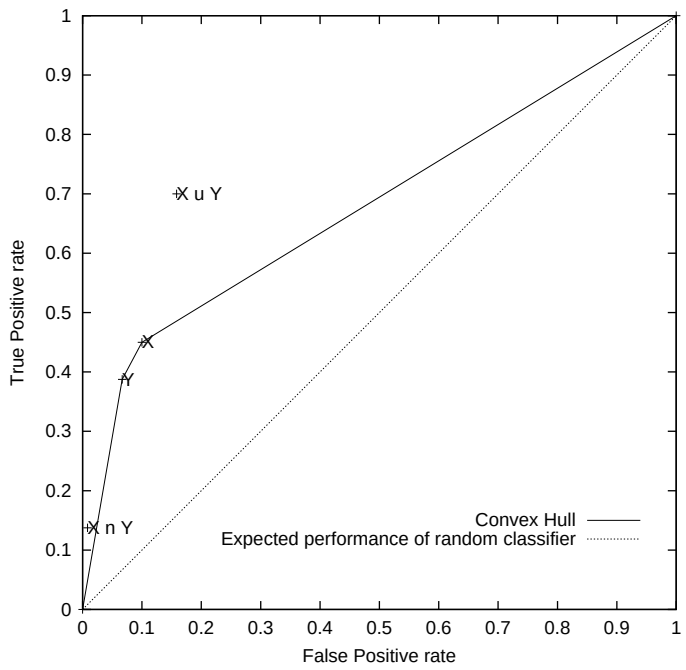

Figure 2: ROC for classifiers shown in Figure 1