Appendix A

Magnetic field calculations

When an object is placed in a homogenous magnetic induction field $\mathbf{B}_0$, a distortion of this field will occur, depending on the susceptibility distribution $\chi(\mathbf{r})$ of the object. Assuming the object is non-conductive (so $J = 0$), the following Maxwell's equations will be satisfied

$$\begin{aligned} \nabla \times \mathbf{H} &= 0 && \text{(A.1)} \\ \nabla \cdot \mathbf{B} &= 0 && \text{(A.2)} \end{aligned}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{B} = \mu\mathbf{H}$ is the total magnetic induction and the permeability, μ, is related to the susceptibility, χ, by

$$\mu = \mu_0(1 + \chi) \tag{A.3}$$

where μ_0 is the permeability of free-space.

Due to time-invariance and the absence of stationary currents the curl of $\mathbf{H}$ vanishes and $\mathbf{H}$ can be expressed as the gradient of the magnetic scalar potential $H = \nabla\phi$. This gives

$$\mathbf{B} = \mu_0(1 + \chi)\nabla\phi \tag{A.4}$$

and hence

$$\mu_0 \, \nabla \cdot (\, (1 + \chi)\nabla\phi) = 0. \tag{A.5}$$

Let the susceptibility, χ, be expanded as

$$\chi = \chi_0 + \delta\chi_1 \tag{A.6}$$

where χ_0 is the susceptibility of air (i.e. $\mu_{air} = \mu_0(1+\chi_0)$ with $\chi_0 = 4 \times 10^{-7}$), and δ is a constant that represents the average difference in the susceptibility of tissue and air (e.g. -9.5×10^{-6} for brain tissues), such that the typical range of χ_1 is from 0 to 1. $\chi_1 = 0$ when only air present in the voxel, $\chi_1 = 1$ when only tissue present in the voxel. The values in between reflect the partial volume susceptibility.

Similarly, ϕ can be expanded in a series

$$\phi = \phi_0 + \delta\phi_1 + \delta^2\phi_2 + \dots \tag{A.7}$$

This perturbation expansion in δ can be substituted back into equation A.5 to give

$$\mu_0(1+\chi_0)\Delta\phi_0 = 0 \tag{A.8}$$

$$(1+\chi_0)\Delta\phi_1 + \nabla\cdot(\chi_1\nabla\phi_0) = 0 \tag{A.9}$$

for the zeroth and first order terms in δ.

Using the zeroth order equation together with standard vector calculus identities gives a 3D Poisson equation

$$\Delta\phi_1 = \frac{-1}{1+\chi_0}\left(\nabla\cdot(\chi_1\nabla\phi_0)\right). \tag{A.10}$$

The Green's function for this equation is

$$G(\mathbf{r}) = \frac{-1}{4\pi\|\mathbf{r}\|}. \tag{A.11}$$

This allows the solution of the Poisson equation to be written as a convolution

$$\phi_1(\mathbf{r}) = \iiint G(\mathbf{r}-\mathbf{r}')f(\mathbf{r}')\,d\mathbf{r}' \tag{A.12}$$

or more consisely as, $\phi_1 = G * f$, where $f = \frac{-1}{1+\chi_0}\left(\nabla \cdot (\chi_1 \nabla \phi_0)\right)$.

From this the z-component of the B field can be written as

$$\begin{aligned} B_z &= \mu H_z = \mu \frac{\partial \phi}{\partial z} \\ &= \mu_0(1+\chi_0)\frac{\partial \phi_0}{\partial z} + \delta\, \mu_0 \left(\chi_1 \frac{\partial \phi_0}{\partial z} + (1+\chi_0)\frac{\partial \phi_1}{\partial z}\right) + O(\delta^2). \end{aligned} \tag{A.13}$$

As the zeroth order term is $B_z^{(0)} = \mu_0(1+\chi_0)\,\partial\phi_0/\partial z$, then the first order term is

$$B_z^{(1)} = \frac{\chi_1}{1+\chi_0} B_z^{(0)} + \mu_0(1+\chi_0)\frac{\partial \phi_1}{\partial z}. \tag{A.14}$$

Using the fact that

$$\frac{\partial}{\partial x}(G * f) = G * \frac{\partial f}{\partial x} = \frac{\partial G}{\partial x} * f$$

holds for any G and f, together with equations A.8, A.9, A.12 and A.14, gives

$$\begin{aligned} B_z^{(1)} \simeq\ & \frac{\chi_1}{1+\chi_0} B_z^{(0)} - \frac{1}{1+\chi_0}\left(\left(\frac{\partial^2 G}{\partial x \partial z}\right) * \left(\chi_1 B_x^{(0)}\right) + \right. \\ & \left. \left(\frac{\partial^2 G}{\partial y \partial z}\right) * \left(\chi_1 B_y^{(0)}\right) + \left(\frac{\partial^2 G}{\partial z^2}\right) * \left(\chi_1 B_z^{(0)}\right)\right) . \end{aligned} \tag{A.15}$$

More detailed explanation and additional calculations are described in [37].

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