

COMP1004 ALGORITHMS & DATA STRUCTURES COURSEWORK 2, 2011

Please hand in to 5th floor reception by 12pm on Monday 21 March

- 1) Are the following statements true or false? Justify your answers using a careful argument based on the formal mathematical definition of 'O' notation. (You may assume where necessary that n is a positive integer.)
- (i) $n^3 \in O(n^2)$
 - (ii) $\log_2(2n) \in O(\log_2(n))$
 - (iii) $2^n \in O(4^n)$
 - (iv) $(n + 1)! \in O(n!)$
- 2) Consider the following short procedures, written in pseudocode. In each case work out $f(n)$, the exact number of unit-time operations the procedure requires as a function of the input size n , simplifying your final answer using O-notation.
- (i)

```
for i <- 1 to n do
  for j <- 2 to (n+i) do
    // a unit cost operation
```
 - (ii)

```
for i <- 1 to n do
  for j <- 1 to n do
    for k <- 1 to (i+j) do
      // a unit cost operation
```
 - (iii)

```
for i <- 1 to n do
  for j <- 1 to n do
    for k <- 1 to i*j do
      // a unit cost operation
```

TURN OVER

3) Solve the following recurrence relations, simplifying your final answer using 'O' notation. (You may assume that n is a power of 2 where appropriate.)

(i) $f(0) = 2$
 $f(n) = 6f(n-1) - 5, n > 0$

(ii) $f(0) = 2$
 $f(1) = 5$
 $f(n) = 5f(n-1) - 6f(n-2), n > 1$

(iii) $f(0) = 3$
 $f(1) = 12$
 $f(n) = 6f(n-1) - 9f(n-2), n > 1$

(iv) $f(1) = 3$
 $f(2) = 9$
 $f(n) = 5f(\frac{n}{2}) - 4f(\frac{n}{4}), n > 2$

4) Consider the following recurrence relations (in which you may consider that the variable n is always positive):

$$f(1) = 1$$
$$f(n) = 4f(\frac{n}{2}), n > 1$$

$$g(0) = 1$$
$$g(n) = 2g(n-1), n > 0$$

(i) Solve each of these recurrence relations as a function of n .

(ii) For what positive integer value(s) of n is the solution for $g(n)$ less than that for $f(n)$?