COMP1004 ALGORITHMS & DATA STRUCTURES COURSEWORK 2, 2011

Please hand in to 5th floor reception by 12pm on Monday 21 March

1) Are the following statements true or false? Justify your answers using a careful argument based on the formal mathematical definition of 'O' notation. (You may assume where necessary that n is a positive integer.)

(i) $n^3 \in O(n^2)$

(ii) $\log_2(2n) \in O(\log_2(n))$

(iii) $2^n \in O(4^n)$

(iv) $(n+1)! \in O(n!)$

2) Consider the following short procedures, written in pseudocode. In each case work out f(n), the exact number of unit-time operations the procedure requires as a function of the input size n, simplifying your final answer using O-notation.

(i)
```
for i <− 1 to n do
  for j <− 2 to (n+i) do
    // a unit cost operation
```

(ii)
```
for i <− 1 to n do
  for j <− 1 to n do
    for k <− 1 to (i+j) do
      // a unit cost operation
```

(iii)
```
for i <− 1 to n do
  for j <− 1 to n do
    for k <− 1 to i*j do
      // a unit cost operation
```

TURN OVER

3) Solve the following recurrence relations, simplifying your final answer using 'O' notation. (You may assume that n is a power of 2 where appropriate.)

(i) $f(0) = 2$
$f(n) = 6f(n-1) - 5, \quad n > 0$

(ii) $f(0) = 2$
$f(1) = 5$
$f(n) = 5f(n-1) - 6f(n-2), \quad n > 1$

(iii) $f(0) = 3$
$f(1) = 12$
$f(n) = 6f(n-1) - 9f(n-2), \quad n > 1$

(iv) $f(1) = 3$
$f(2) = 9$
$f(n) = 5f(n/2) - 4f(n/4), \quad n > 2$

4) Consider the following recurrence relations (in which you may consider that the variable n is always positive):

$f(1) = 1$
$f(n) = 4f(n/2), \quad n > 1$

$g(0) = 1$
$g(n) = 2g(n-1), \quad n > 0$

(i) Solve each of these recurrence relations as a function of n.

(ii) For what positive integer value(s) of n is the solution for g(n) less than that for f(n)?