Robust Probabilistic Projections: Errata Appendix

Equation (47) should read as follows:

$$\begin{aligned}
\mathbf{V}_1\boldsymbol{\Upsilon}^2\mathbf{V}_1^{\mathrm{T}} &= (\mathbf{V}_1\boldsymbol{\Upsilon}\mathbf{V}_2^{\mathrm{T}})(\mathbf{V}_1\boldsymbol{\Upsilon}\mathbf{V}_2^{\mathrm{T}})^{\mathrm{T}} \\
&= \bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}}\bar{\boldsymbol{\Sigma}}_{12}\bar{\boldsymbol{\Sigma}}_{22}^{-1}\bar{\boldsymbol{\Sigma}}_{21}\bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}} \\
&= \bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}}\widehat{\mathbf{W}}_1\widehat{\mathbf{W}}_2^{\mathrm{T}}(\widehat{\mathbf{W}}_2\widehat{\mathbf{W}}_2^{\mathrm{T}} + \boldsymbol{\Psi}_2^{-1})^{-1}\widehat{\mathbf{W}}_2\widehat{\mathbf{W}}_1^{\mathrm{T}}\bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}} \\
&= \bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}}\widehat{\mathbf{W}}_1(\mathbf{I}_d - \mathbf{B}_2^{-1})\widehat{\mathbf{W}}_1^{\mathrm{T}}\bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}} \qquad (1) \\
&= \widetilde{\mathbf{V}}_1(\mathbf{I}_d - \mathbf{B}_1^{-1})^{1/2}(\mathbf{I}_d - \mathbf{B}_2^{-1})(\mathbf{I}_d - \mathbf{B}_1^{-1})^{1/2}\widetilde{\mathbf{V}}_1^{\mathrm{T}} \\
&= \widetilde{\mathbf{V}}_1\mathbf{R}_1\widetilde{\boldsymbol{\Upsilon}}^2\mathbf{R}_1^{\mathrm{T}}\widetilde{\mathbf{V}}_1^{\mathrm{T}}
\end{aligned}$$

where we made use of the Woodburry inversion formula in (1) twice. We also defined $\mathbf{B}_i \equiv \widehat{\mathbf{W}}_i^{\mathrm{T}}\boldsymbol{\Psi}_i\widehat{\mathbf{W}}_i + \mathbf{I}_d$ and $\widetilde{\mathbf{V}}_1 \equiv \bar{\boldsymbol{\Sigma}}_{11}^{-\frac{1}{2}}\widehat{\mathbf{W}}_1(\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}}$. The latter is an orthogonal matrix, since we have

$$\begin{aligned}
\widetilde{\mathbf{V}}_1^{\mathrm{T}}\widetilde{\mathbf{V}}_1 &= (\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}}\widehat{\mathbf{W}}_1^{\mathrm{T}}\bar{\boldsymbol{\Sigma}}_{11}^{-1}\widehat{\mathbf{W}}_1(\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}} \\
&= (\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}}(\mathbf{I}_d - \mathbf{B}_1^{-1})(\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}} \\
&= \mathbf{I}_d.
\end{aligned}$$

The matrix $\mathbf{R}_1$ contains the eigenvectors of $(\mathbf{I}_d - \mathbf{B}_1^{-1})^{1/2}(\mathbf{I}_d - \mathbf{B}_2^{-1})(\mathbf{I}_d - \mathbf{B}_1^{-1})^{1/2}$ and $\widetilde{\boldsymbol{\Upsilon}}^2$ the corresponding eigenvalues. Similarly, $\mathbf{R}_2$ contains the eigenvectors of $(\mathbf{I}_d - \mathbf{B}_2^{-1})^{1/2}(\mathbf{I}_d - \mathbf{B}_1^{-1})(\mathbf{I}_d - \mathbf{B}_2^{-1})^{1/2}$ and the same eigenvalues $\widetilde{\boldsymbol{\Upsilon}}^2$.

Identifying the first and the last equalities of (47), we find $\mathbf{V}_1 = \widetilde{\mathbf{V}}_1\mathbf{R}_1$ and $\boldsymbol{\Upsilon} = \widetilde{\boldsymbol{\Upsilon}}$. Doing the same development for $\mathbf{V}_2\boldsymbol{\Upsilon}^2\mathbf{V}_2^{\mathrm{T}}$, one gets $\mathbf{V}_2 = \widetilde{\mathbf{V}}_2\mathbf{R}_2$. Hence, we find

$$\begin{cases}
\mathbf{U}_{1d} = \bar{\boldsymbol{\Sigma}}_{11}^{-1}\widehat{\mathbf{W}}_1(\mathbf{I}_d - \mathbf{B}_1^{-1})^{-\frac{1}{2}}\mathbf{R}_1, \\
\mathbf{U}_{2d} = \bar{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\mathbf{W}}_2(\mathbf{I}_d - \mathbf{B}_2^{-1})^{-\frac{1}{2}}\mathbf{R}_2.
\end{cases}$$