# Approximate system identification: Misfit versus latency

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Two fundamentally different approaches in system identification, which are used for quantification of the model–data mismatch, are misfit and latency. The aim of this talk is to explain the rationale behind them and link them to statistical estimation methods—errors-in-variables regression and classical regression—respectively.

Consider a given model and observed data. The model $\mathscr{B}$ is viewed as a set of possible outcomes from the set $\mathscr{U}$ of all conceivable outcomes, and the data $w$ is an element of $\mathscr{U}$. The lack of fit, called misfit, between the data $w$ and the model $\mathscr{B}$ (with respect to a given norm in $\mathscr{U}$) is the distance from $w$ to $\mathscr{B}$. The misfit is the norm of the smallest modification $\Delta w$ of the data that renders the modified data $\widehat{w} := w + \Delta w$ exact for the model, i.e., $\widehat{w} \in \mathscr{B}$.

The latency approach augments the model $\mathscr{B}$ by an extra variable $e$, called latent variable. Let $\mathscr{B}_{\text{aug}}$ be the augmented model $\{ (w,e) \mid w \in \mathscr{B} \}$. The lack of fit, called latency, between the data $w$ and the model $\mathscr{B}$ is then the size (with respect to a given norm) of the smallest latent variable $\widehat{e}$ that makes the augmented data vector $(w,\widehat{e})$ consistent with the augmented model, i.e., $(w,\widehat{e}) \in \mathscr{B}_{\text{aug}}$.

Approximate system identification methods that minimize the misfit and latency criteria (over a parameterized set of models) correspond, respectively, to the maximum likelihood method in the errors-in-variables setting (i.e., all variables are assumed noise corrupted), and to the prediction error identification method in the classical stochastic setting (i.e., system driven by white noise). Therefore, the errors-in-variables setting and the classical stochastic setting provide statistical justification for the minimization of the misfit and latency criteria as consistent and efficient estimators under suitable assumptions. Conversely, misfit and latency minimization provide a deterministic interpretation of the stochastic estimation methods as approximate identification methods.

## References

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