User Authentication and Cryptographic Primitives

Brad Karp
UCL Computer Science

CS GZ03 / M030

16th November 2015

Outline

- Authenticating users
  - Local users: hashed passwords
  - Remote users: s/key
  - Unexpected covert channel: the Tenex password-guessing attack
- Symmetric-key-cryptography
- Public-key cryptography usage model
- RSA algorithm for public-key cryptography
  - Number theory background
  - Algorithm definition

Dictionary Attack on Hashed Password Databases

- Suppose hacker obtains copy of password file (until recently, world-readable on UNIX)
- Compute H(x) for 50K common words
- String compare resulting hashed words against passwords in file
- **Learn all users' passwords that are common English words after only 50K computations of H(x)!**
- **Same hashed dictionary works on all password files in world!**

Salted Password Hashes

- Generate a random string of bytes, r
- For user password x, store [H(r,x), r] in password file
- Result: same password produces different result on every machine
  - So must see password file before can hash dictionary
  - …and single hashed dictionary won't work for multiple hosts
- Modern UNIX: password hashes salted; hashed password database readable only by root

Salted Password Hashes

- Generate a random string of bytes, r

Dictionary attack still possible after attacker sees password file!

Users should pick passwords that aren't close to dictionary words.

  - So must see password file before can hash dictionary
  - …and single hashed dictionary won't work for multiple hosts
- Modern UNIX: password hashes salted; hashed password database readable only by root

Tenex Password Attack: An Information Leak

- Tenex OS stored directory passwords in cleartext
- OS supported system call:
  - pw_validate(directory, pw)
- Implementation simply compared pw to stored password in directory, char by char
- Clever attack:
  - Make pw span two VM pages, put 1st char of guess in first page, rest of guess in second page
  - See whether get a page fault—if not, try next value for 1st char, &c.; if so, first char correct!
  - Now position 2nd char of guess at end of 1st page, &c.
  - **Result: guess password in time linear in length!**

Tenex Password Attack: An Information Leak

- Tenex OS stored directory passwords in cleartext

Lessons:

Don’t store passwords in cleartext.

Information leaks are real, and can be extremely difficult to find and eliminate.

- Clever attack:
  - Make pw span two VM pages, put 1st char of guess in first page, rest of guess in second page
  - See whether get a page fault—if not, try next value for 1st char, &c.; if so, first char correct!
  - Now position 2nd char of guess at end of 1st page, &c.
  - **Result: guess password in time linear in length!**

Remote User Authentication

- Consider the case where Alice wants to log in remotely, across LAN or WAN from server
- Suppose network links can be eavesdropped by adversary, Eve
- Want scheme immune to replay: if Eve overhears messages, shouldn't be able to log in as Alice by repeating them to server
- Clear non-solutions:
  - Alice logs in by sending {alice, password}
  - Alice logs in by sending {alice, H(password)}

Remote User Authentication (2)

- Desirable properties:
  - Message from Alice must change unpredictably at each login
  - Message from Alice must be verifiable at server as matching secret value known only to Alice
- Can we achieve these properties using **only** a cryptographic hash function?

Remote User Authentication: s/key

- Denote by $H^n(x)$ n successive applications of cryptographic hash function H() to x
  - i.e., $H^3(x) = H(H(H(x)))$
- Store in server's user database:

  `alice:99:`H^{99}`(password)`
- At first login, Alice sends:

  {alice, H^{98}(password)}
- Server then updates its database to contain:

  `alice:98:`H^{98}`(password)`
- At next login, Alice sends:

  {alice, H^{97}(password)}
  - and so on…

Properties of s/key

- Just as with any hashed password database, Alice must store her secret on the server securely (best if physically at server's console)
- Alice must choose total number of logins at time of storing secret
- When logins all "used", must store new secret on server securely again

Secrecy through Symmetric Encryption

- Two functions: E() encrypts, D() decrypts
- Parties share secret key K
- For message M:
  - E(K, M) → C
  - D(K, C) → M
- M is plaintext; C is ciphertext
- Goal: attacker cannot derive M from C without K

Idealized Symmetric Encryption: One-Time Pad

- Secretly share a truly random bit string P at sender and receiver
- Define $\oplus$ as bit-wise XOR
- $C = E(M) = M \oplus P$
- $M = D(C) = C \oplus P$
- Use bits of P only once; **never use them again!**

Stream Ciphers: Pseudorandom Pads

- Generate pseudorandom bit sequence (stream) at sender and receiver from short key
- Encrypt and decrypt by XOR'ing message with sequence, as with one-time pad
- Most widely used stream cipher: RC4
- Again, **never, ever re-use bits from pseudorandom sequence!**
- What's wrong with reusing the stream?
  - Alice → Server: c_1 = E(s, "Visa card number")
  - Server → Alice: c_2 = E(s, "Transaction confirmed")
  - Suppose Eve hears both messages
  - Eve can compute:
    $m = c_1 \oplus c_2 \oplus$ "Transaction confirmed"

Symmetric Encryption: Block Ciphers

- Divide plaintext into fixed-size blocks (typically 64 or 128 bits)
- Block cipher maps each plaintext block to same-length ciphertext block
- Best today to use AES (others include Blowfish, DES, …)
- Of course, message of arbitrary length; how to encrypt message of more than one block?

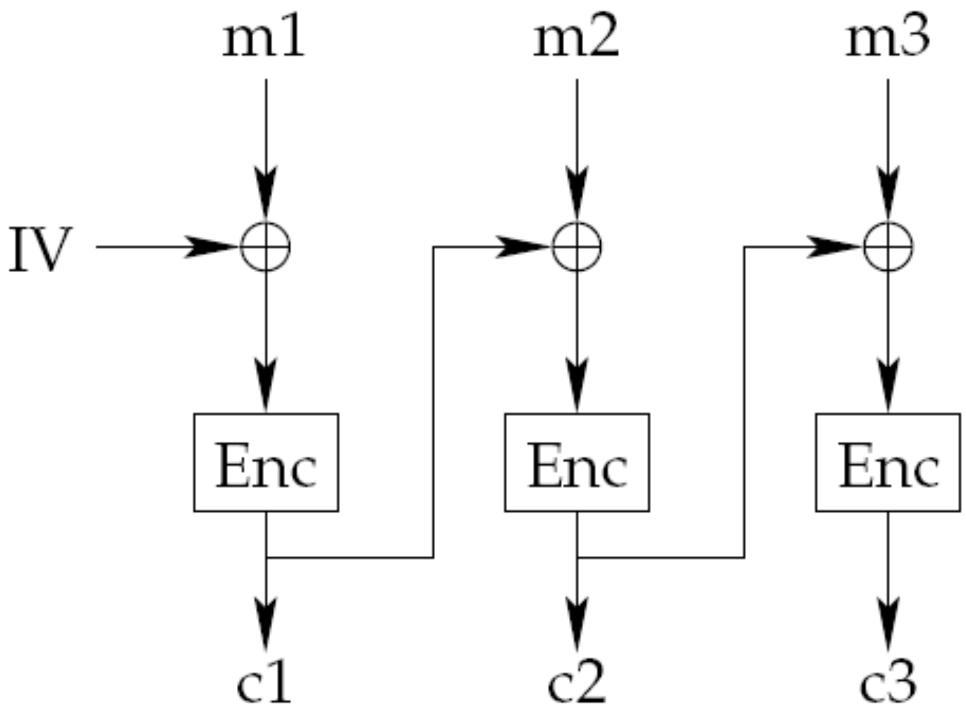
Using Block Ciphers: ECB Mode

- Electronic Code Book method
- Divide message M into blocks of cipher's block size
- Simply encrypt each block individually using the cipher
- Send each encrypted block to receiver
- Presume cipher provides secrecy, so attacker cannot decrypt any block
- **Does ECB mode provide secrecy?**

Avoid ECB Mode!

- ECB mode **does not provide robust secrecy!**
- What if there are repeated blocks in the plaintext? Repeated as-is in ciphertext!
- What if sending sparse file, with long runs of zeroes? Non-zero regions obvious!
- WW II U-Boat example (Bob Morris):
  - Each day at same time, when no news, send encrypted message: “Nichts zu melden.”
  - When there’s news, send the news at that time.
  - Obvious when there’s news
  - Many, many ciphertexts of same known plaintext made available to adversary for cryptanalysis—a worry even if encryptions of same plaintext produce different ciphertexts!

Using Block Ciphers: CBC Mode



- Better plan: make encryptions of successive blocks depend on one another, and initialization vector known to receiver

Integrity with Symmetric Crypto: Message Authentication Codes

- How does receiver know if message modified en route?
- Message Authentication Code:
  - Sender and receiver share secret key K
  - On message M, v = MAC(K, M)
  - Attacker cannot produce valid {M, v} without K
- Append MAC to message for tamper-resistance:
  - Sender sends {M, MAC(K, M)}
  - M could be ciphertext, M = E(K', m)
  - Receiver of {M, v} can verify that v = MAC(K, M)
- Beware replay attacks—replay of prior {M, v} by Eve!

HMAC: A MAC Based on Cryptographic Hash Functions

- HMAC(K, M) =
  $H(K \oplus opad \,.\, H(K \oplus ipad \,.\, M))$
- where:
  - . denotes string concatenation
  - opad = 64 repetitions of 0x36
  - ipad = 64 repetitions of 0x5c
  - H() is a cryptographic hash function, like SHA-256
- Fixed-size output, even for long messages

Public-Key Encryption: Interface

- Two keys:
  - Public key: K, published for all to see
  - Private (or secret) key: K^{-1}, kept secret
- Encryption: $E(K, M) \rightarrow \{M\}_K$
- Decryption: $D(K^{-1}, \{M\}_K) \rightarrow M$
- Provides secrecy, like symmetric encryption:
  - Can't derive M from $\{M\}_K$ without knowing K^{-1}
- Same public key used by all to encrypt all messages to same recipient
  - Can't derive K^{-1} from K

Number Theory Background: Modular Arithmetic Primer (1)

- Recall the “mod” operator: returns remainder left after dividing one integer by another, the modulus
  - e.g., 15 mod 6 = 3
- That is:

  $a \bmod n = r$

  which just means

  $a = kn + r$ for some integers k and r
- Note that $0 <= r < n$

Modular Arithmetic Primer (2)

- In modular arithmetic, constrain range of integers to be only the residues [0, n-1], for modulus n
  - e.g., (12 + 13) mod 24 = 1
  - We may also write $12 + 13 \equiv 1 \pmod{24}$
- Modular arithmetic retains familiar properties: commutative, associative, distributive
- Same results whether mod taken at each arithmetic operation, or only at end, e.g.:

  (a + b) mod n = ((a mod n) + (b mod n)) mod n

  (ab) mod n = (a mod n)(b mod n) mod n

Modular Arithmetic: Advantages

- Limits precision required: working mod n, where n is k bits long, any single arithmetic operation yields at most 2k bits
  - …so results of even seemingly expensive ops, like exponentiation (a^x) fit in same number of bits as original operand(s)
  - Lower precision means faster arithmetic
- Some operations in modular arithmetic are computationally very difficult:
  - e.g., computing discrete logarithms: find integer x s.t. $a^x \equiv b \pmod{n}$

Modular Arithmetic: Advantages

- Limits precision required: working mod n, where n is k bits long, any single arithmetic operation yields at most 2k bits

Cryptography leverages "difficult" operations; want reversing encryption without key to be computationally intractable!

- Some operations in modular arithmetic are computationally very difficult:
  - e.g., computing discrete logarithms: find integer x s.t. $a^x \equiv b \pmod{n}$

Modular Arithmetic: Inverses (1)

- In real arithmetic, every integer has a multiplicative inverse—its reciprocal—and their product is 1
  - e.g., $7x = 1 \rightarrow x = (1/7)$
- What does an inverse in modular arithmetic (say, mod 11) look like?

$$7x \equiv 1 \pmod{11}$$

  - that is, $7x = 11k + 1$ for some x and k
  - so $x = 8$ (where $k = 5$)

Aside: Prime Numbers

- Recall: prime number is integer > 1 that is evenly divisible only by 1 and itself
- Two integers a and b are relatively prime if they share no common factors but 1; i.e., if gcd(a, b) = 1
- There are infinitely many primes
- Large primes (512 bits and longer) figure prominently in public-key cryptography

Modular Arithmetic: Inverses (2)

- In general, finding modular inverse means finding x s.t. $a^{-1} \equiv x \pmod{n}$
- **Does modular inverse always exist?**
  - No! Consider $2^{-1} \equiv x \pmod{8}$
- In general, when a and n are relatively prime, modular inverse x exists and is unique
- When a and n not relatively prime, x doesn't exist
- When n prime, all of [1…n-1] relatively prime to n, and have an inverse in that range

Modular Arithmetic: Inverses (2)

- In general, finding modular inverse means

Algorithm to find modular inverse: extended Euclidean Algorithm. Tractable; requires O(log n) divisions.

- In general, when a and n are relatively prime, modular inverse x exists and is unique
- When a and n not relatively prime, x doesn't exist
- When n prime, all of [1…n-1] relatively prime to n, and have an inverse in that range

Euler's Phi Function: Efficient Modular Inverses on Relative Primes

- $\varphi(n)$ = number of integers < n that are relatively prime to n
- If n prime, $\varphi(n) = n-1$
- If $n=pq$, where p and q prime: $\varphi(n) = (p-1)(q-1)$
- If a and n relatively prime, Euler's generalization of Fermat's little theorem:

  $a^{\varphi(n)} \bmod n = 1$

- and thus, to find inverse x s.t. $x = a^{-1} \bmod n$:

  $x = a^{\varphi(n)-1} \bmod n$

RSA Algorithm (1)

- [Rivest, Shamir, Adleman, 1978]
- Recall that public-key cryptosystems use two keys per user:
  - K, the public key, made available to all
  - K^{-1}, the private key, kept secret by user

RSA Algorithm (2)

- Choose two random, large primes, p and q, of equal length, and compute $n=pq$
- Randomly choose encryption key e, s.t. e and $(p-1)(q-1)$ are relatively prime
- Use extended Euclidean algorithm to compute d, s.t. $d = e^{-1} \bmod ((p-1)(q-1))$
- Public key: $K = (e, n)$
- Private key: $K^{-1} = d$
- Discard p and q

RSA Algorithm (3)

- Encryption:
  - Divide message M into blocks m_i, each shorter than n
  - Compute ciphertext blocks c_i with:
    $c_i = m_i^e \bmod n$
- Decryption
  - Recover plaintext blocks m_i with:
    $m_i = c_i^d \bmod n$

Why Does RSA Decryption Recover Original Plaintext?

- Observe that $c_i^d = (m_i^e)^d = m_i^{ed}$
- Note that $ed \equiv 1 \pmod{(p-1)(q-1)}$
  because e and d are inverses mod (p-1)(q-1)
- So:

  $ed \equiv 1 \pmod{(p-1)}$, and thus ed = k(p-1)+1
  $ed \equiv 1 \pmod{(q-1)}$, and thus ed = h(q-1)+1
- Consider case where m_i and p are relatively prime:

  $m_i^{(p-1)} \equiv 1 \pmod{p}$ by Euler's generalization of Fermat's little theorem
  - so $m_i^{ed} = m_i^{k(p-1)+1} = m_i(m_i^{(p-1)})^k \equiv m_i \pmod{p}$
- And case where m_i a multiple of p:

  $m_i^{ed} = 0^{ed} = 0 \equiv m_i \pmod{p}$
- Thus in all cases, $m_i^{ed} \equiv m_i \pmod{p}$

Why Does RSA Decryption Recover Original Plaintext? (2)

- Similarly, $m_i^{ed} \equiv m_i \pmod{q}$
- Now:

$$m_i^{ed} - m_i \equiv 0 \pmod{p}$$
$$m_i^{ed} - m_i \equiv 0 \pmod{q}$$

- Because p, q both prime and distinct:

$$m_i^{ed} - m_i \equiv 0 \pmod{(pq)}$$

- So $c_i^d = m_i^{ed} \equiv m_i \pmod{n}$