Taking the relative significance of inconsistencies into account

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Abstract. Inconsistencies can arise between a news report and domain knowledge. Some of these inconsistencies may be more significant than others, and some news reports may contain more inconsistencies than others. This creates problems of deciding whether to act on these inconsistencies, and if so how. Having some understandering of the "degree of inconsistency" of a news report can help in deciding how to act on it. Moreover, inconsistencies between information in a news report and domain knowledge can tell us important things about the news report. For this we introduce the notion of a significance function that gives a value in the unit interval for each possible inconsistency that can arise in a news report. To illustrate, consider a news report on a football match, where an inconsistency in the final score of the match could be assigned significance 1, whereas an inconsistency in the name of the player who scored the most goals in less significant and could be assigned a value below 1. How we define the significance function depends on the application. To help define a significance function, we introduce rankings over the types of information that can appear in the news reports in the application. In this paper, we present a general framework for significance functions and use news reports as an illustration of an application.

1 Introduction

A simple approach to measuring the degree of inconsistency in a database (a set of formulae) is to consider whether there are any minimally inconsistent subsets of the database. Each minimally inconsistent subset in a database is regarded as grounds for rejecting the database. Yet some of the inconsistencies might not be so important as to cause the database to be unusable.

To address this, we downgrade the value of the significance of some inconsistencies to a value less than 1 but greater than 0. So if an inconsistency has significance less than 1, we regard it as not being sufficient grounds for rejecting the report, whereas if an inconsistency has a significance equal to 1, then it is sufficient for rejecting the report. Now, if we have a few inconsistencies, then each might not be sufficient grounds for rejecting the report, but taken together they might be sufficient grounds for rejecting the report. So the value assigned is meant to capture the degree to which futher inconsistencies may be added to it to cause rejection.

So our framework for measuring the degree of inconsistency of a database on (1) the number of minimal inconsistencies and (2) the significance of each of these inconsistencies. This leads us to assessment of the usability of the database. If it is inconsistent to a degree less than 1, then the inconsistencies can be ignored, and if it is inconsistent to a degree greater than or equal to 1, then some action needs to be invoked. For some general discussion of acting on inconsistency see [GH91,GH93].

As far as we are aware, there are no proposals for measuring the degree of inconsistency in a set of formulae, and in particular for taking the relative significance of inconsistencies into account. Some approaches touch on the topic. In diagnostic systems, there are proposals that offer preferences for certain kinds of consistent subsets of inconsistent information [KW87,Rei87], in proposals for belief revision, epistemic entrenchment is an ordering over formulae which reflects the preference for which formulae to give up in case of inconsistency [Gar88], in proposals for drawing inferences from inconsistent information there is a preference for inferences from certain consistent subsets (E.g. [MR70,Bre89,BDP93,CRS93,Cho95,EGH95]), in proposals for approximating entailment, two sequences of entailment relation are defined (the first is sound but not complete, and the second is complete but not sound) which converge to classical entailment [Lev84,SC95], and in proposals for partial consistency checking, checking is terminated after the search space exceeds a threshold which gives a measure of partial consistency of the data (E.g. Maximum generalized satisfiability [Pap94]). However, none of these proposals provide a definition for degree of inconsistency.

Inconsistency handling is a big problem in computer science and IT. Techniques for measuring inconsistencies in information should be important instruments in helping to address this problem. Here we present a new framework for measuring inconsistency, and use the application area of news reports to illustrate the approach.

2 Basic definitions

We start with a few definitions for classical logic.

Definition 1. *Let $\mathcal{L}$ be the set of* **classical propositional formulae** *formed from a set of atoms, and the logical connectives $\{\vee, \wedge, \neg, \rightarrow\}$. Let $\perp$ denote any inconsistent formula.*

Definition 2. *Let $\Gamma \in \wp(\mathcal{L})$, $Con(\Gamma) = \{\Phi \subseteq \Gamma \mid \Phi \not\vdash \perp\}$, and $Incon(\Gamma) = \{\Phi \subseteq \Gamma \mid \Phi \vdash \perp\}$.*

$$MC(\Gamma) = \{\Phi \in Con(\Gamma) \mid \forall \Psi \in Con(\Gamma)\ \Phi \not\subset \Psi\}$$

$$MI(\Gamma) = \{\Phi \in Incon(\Gamma) \mid \forall \Psi \in Incon(\Gamma)\ \Psi \not\subset \Phi\}$$

We call $MI(\Gamma)$ the set of minimally inconsistent subsets of Γ, and $MC(\Gamma)$ the set of maximally consistent subsets of Γ.

Definition 3. *Let Δ be a set of first-order classical logic formulae which will be the* **domain knowledge** *for the news reports. In this paper, we assume Δ is always consistent.*

3 Representing news reports

A news report is just a set of literals as defined below.

Definition 4. *A* **news report** *is a set of binary predicates where r is the report number, $t_1, .., t_n$ are terms, and $p_1, .., p_n$ are the predicate symbols:*

$$p_1(r, t_1), ..., p_n(r, t_n)$$

The set of news reports is denoted R.

Each term is used to represent a piece of information in a news report and may be equivalent to a word, or number, or a simple phrase. The predicate symbol is used to tag or categorise each piece of information.

Example 1. The following is a news report:

$$\{\texttt{date(r1, 01Jan2001)}, \texttt{city(r1, London)}, \texttt{precipitation(r1, rain)}\}$$

A potential inconsistency that can arise in a news report is any set of literals that may be rebutted by the domain knowledge.

Definition 5. *Let Φ be a news report and let Δ be domain knowledge. For $\{\phi_1, .., \phi_n\} \subseteq \Phi$, $\{\phi_1, .., \phi_n\}$ is* **rebutted** *by Δ iff $\Delta \vdash \neg(\phi_1 \wedge \ldots \wedge \phi_n)$.*

We represent a potential inconsistency in a report by a set of literals of that report. Since we start without any preconceptions about the domain knowledge, potentially any set of literals may be rebutted by the domain knowledge, and so any non-empty subset of a news report is a potential inconsistency.

Definition 6. *The* **set of potential inconsistencies** *for a news report $\Phi \in R$ is $\wp(\Phi)\backslash\emptyset$. The set of all potential inconsistencies for R is $P_R = \{X \mid X \in \wp(\Phi)\backslash\emptyset$ and $\Phi \in R\}$.*

Example 2. Continuing Example 1, the set of potential inconsistencies for this report are:

$$\begin{gathered}\{\{\texttt{date(r1, 01Jan2001)}, \texttt{city(r1, London)}, \texttt{precipitation(r1, rain)}\},\\ \{\texttt{date(r1, 01Jan2001)}, \texttt{city(r1, London)}\},\\ \{\texttt{date(r1, 01Jan2001)}, \texttt{precipitation(r1, rain)}\},\\ \{\texttt{city(r1, London)}, \texttt{precipitation(r1, rain)}\},\\ \{\texttt{date(r1, 01Jan2001)}\},\\ \{\texttt{city(r1, London)}\},\\ \{\texttt{precipitation(r1, rain)}\}\}\end{gathered}$$

Note, a potential inconsistency only involves a single news report. We refer to such an inconsistency as an intra-report inconsistency. We do not consider in this paper any inconsistency that arises between two or more news reports (an inter-report inconsistency).

Any news report may have more than one potential inconsistency. In other words, each news report may have multiple inconsistencies as defined as below.

Definition 7. *The* **set of multiple inconsistencies** *for a news report* $\Phi \in R$ *is* $\wp(\wp(\Phi)\backslash\emptyset)$.

So a multiple inconsistency is a set of potential inconsistencies.

Example 3. Continuing Example 2, an example of a multiple inconsistency is:

$$\{\{\texttt{date}(\texttt{r1}, \texttt{01Jan2001}), \texttt{city}(\texttt{r1}, \texttt{London})\},$$
$$\{\texttt{date}(\texttt{r1}, \texttt{01Jan2001}), \texttt{precipitation}(\texttt{r1}, \texttt{rain})\},$$
$$\{\texttt{precipitation}(\texttt{r1}, \texttt{rain})\}\}$$

Definition 8. *The* **inconsistency space** *for R is* $Q_R = \{X \mid X \in \wp(\wp(\Phi)\backslash\emptyset)$ *and* $\Phi \in R\}$.

The inconsistency space for R is the set of all multiple inconsistencies for R.

Example 4. Consider a news report $\Phi = \{\phi_1, \phi_2\}$. Let $\pi_1 = \{\phi_1\}$, $\pi_2 = \{\phi_2\}$, and $\pi_3 = \{\phi_1, \phi_2\}$. So if $R = \{\Phi\}$, then $P_R = \{\pi_1, \pi_2, \pi_3\}$ and Q_R is the following set:

$$\{\{\}, \{\pi_1\}, \{\pi_2\}, \{\pi_3\}, \{\pi_1, \pi_2, \pi_3\}, \{\pi_1, \pi_2\}, \{\pi_1, \pi_3\}, \{\pi_2, \pi_3\}\}$$

In the inconsistency space, we only consider multiple inconsistencies that contain potential inconsistencies from the same news report. In other words, we do not consider any multiple inconsistencies that contain potential inconsistencies from two different news reports. So we can see that $Q_R \subset \wp(P_R)$ holds.

4 Significance functions

We now consider a type of function for measuring inconsistencies arising in news reports. Our aim is to provide an alternative to using the MI function (given in Definition 2) for characterising inconsistency, and in particular, we seek an alternative to $|MI(\Phi \cup \Delta)|$ where Φ is a news report and Δ is domain knowledge. These alternative functions are called significance functions because they can take into account the significance of the inconsistencies in a news report when assigning the degree of inconsistency of the report.

Definition 9. *A* **significance function** *for a set of reports R is a monotonic function* $M_R : Q_R \mapsto \mathbb{N}$ *such that* $M_R(\emptyset) = 0$.

The notion of a significance function is very general so we need to consider some appropriate constraints for applications.

Definition 10. *A significance function* M_R *is* **additive** *iff for all* $\{\pi_1, .., \pi_n\} \in Q_R$ *the following holds.*

$$M_R(\{\pi_1, .., \pi_n\}) = M_R(\{\pi_1\}) + .. + M_R(\{\pi_n\})$$

Definition 11. *A significance function* M_R *is* **capped** *iff for all* $\pi \in P_R$, $M_R(\{\pi\}) \leq 1$

The capped constraint will be used for all our significance functions. It is quite a natural constraint, and in terms of expressibility, there does not appear to be any advantage in allowing individual inconsistencies to be assigned a value greater than 1.

In order to focus on inconsistencies of importance, we require the following definition for coherence for multiple inconsistencies. Essentially, a multiple inconsistency is coherent if it does not contain supersets of potential inconsistencies.

Definition 12. *Let* R *be a set of reports. For all* $\lambda \in Q_R$, λ *is* **coherent** *iff* $\neg\exists\pi_i, \pi_j \in \lambda$ *such that* $\pi_i \subset \pi_j$.

Focussing on multiple inconsistencies that are coherent eliminates a form of redundancy. It allows us to identify the minimal subsets of a news report that are rebutted by the domain knowledge.

Definition 13. *A significance function* M_R *is a* **classical significance function** *iff for each* $\lambda \in Q_R$, *if* λ *is coherent, then* $M_R(\lambda) = |\lambda|$.

With the classical significance function, each inconsistency is regarded equally and any inconsistency is sufficient grounds for acting on the news report.

Proposition 1. *If* M_R *is a classical significance function for* Q_R, *then (1)* M_R *is capped and (2)* M_R *is additive. The converse does not hold.*

Proof. For each $\lambda \in Q_R$, if λ is coherent, then $M_R(\lambda) = |\lambda|$. So for each $\{\pi\} \in Q_R$, $\{\pi\}$ is coherent, and $M_R(\{\pi\}) = 1$, and hence M_R is capped. Furthermore, for each $\{\pi_1, .., \pi_n\} \in Q_R$, if $\{\pi_1, .., \pi_n\}$ is coherent, then $M_R(\{\pi_1, .., \pi_n\}) = |\{\pi_1, .., \pi_n\}| = M_R(\{\pi_1\}) + .. + M_R(\{\pi_n\})$, and hence M_R is additive. For the converse, consider a capped and additive significance function M'_R where for each $\{\pi\} \in Q_R$, $M'_R(\{\pi\}) < 1$.

The classical significance function applied to a news report is closely related to taking the cardinality of the MI function (Definition 2) applied to the union of the news report and domain knowledge. We formalize this in Proposition 10.

In this paper, we are primarily interested in non-classical significance functions. In particular, we are interested in supporting the downgrading of some potential inconsistencies so they are assigned a value less than 1.

Definition 14. *Let* M_R *be a significance function. A* **downgraded inconsistency** *is a potential inconsistency* $\pi \in P_R$ *such that* $M_R(\{\pi\}) < 1$.

Any downgraded inconsistency is regarded as not being sufficient grounds for rejecting a news report. A significance function is discounting if it supports downgraded inconsistencies.

Definition 15. *A significance function M_R is* **discounting** *iff there is a $\pi \in P_R$ such that $M_R(\{\pi\}) < 1$.*

For some significance functions, there are multiple inconsistencies that take a value less than 1. This is illustrated in the following example.

Example 5. Let M_R be an additive discounting significance function. Let π_1, π_2, and π_3 be potential inconsistencies for a news report where $M_R(\{\pi_1\}) = M_R(\{\pi_2\}) = M_R(\{\pi_3\}) = 0.4$. So

$$M_R(\{\pi_1, \pi_2\}) = M_R(\{\pi_1, \pi_3\}) = M_R(\{\pi_2, \pi_3\}) = 0.8$$
$$M_R(\{\pi_1, \pi_2, \pi_3\}) = 1.2$$

Our intuition is that once a multiple inconsistency is found in a news report that is greater than or equal to 1, then the news report is unusable without some action. Maybe that action involves seeking clarification, or maybe it involves ignoring some of the domain knowledge.

Smaller inconsistencies can be regarded as more serious than bigger inconsistencies because a smaller inconsistency is rebutted by a smaller disjunction of negated literals that are inferred from the domain knowledge — in other words the conflict is more focussed. For example, if a news report contains the literals ϕ_1, ϕ_2, and ϕ_3, and the domain knowledge implies $\neg\phi_1 \vee \neg\phi_2 \vee \neg\phi_3$, then this is less significant than if the domain knowledge implies $\neg\phi_1$.

Definition 16. *A significance function M_R for Q_R is* **focussed** *iff $\forall\{\pi_i\}, \{\pi_j\} \in Q_R$, if $\pi_i \subset \pi_j$ then $M_R(\{\pi_j\}) < M_R(\{\pi_i\})$.*

Example 6. Consider a news report $\Phi = \{\phi_1, \phi_2\}$. Let $\pi_1 = \{\phi_1\}$, $\pi_2 = \{\phi_2\}$, and $\pi_3 = \{\phi_1, \phi_2\}$. So if $R = \{\Phi\}$, then Q_R is

$$\{\{\}, \{\pi_1\}, \{\pi_2\}, \{\pi_3\}, \{\pi_1, \pi_2\}, \{\pi_1, \pi_3\}, \{\pi_2, \pi_3\}, \{\pi_1, \pi_2, \pi_3\}\}$$

An example of a significance function for Q_R is:

$$\begin{array}{ccc} M_R(\{\pi_1\}) = 0.8 & M_R(\{\pi_2\}) = 0.5 & M_R(\{\pi_3\}) = 0.3 \\ M_R(\{\pi_1, \pi_2\}) = 1.3 & M_R(\{\pi_1, \pi_3\}) = 1.1 & M_R(\{\pi_2, \pi_3\}) = 0.8 \\ & M_R(\{\pi_1, \pi_2, \pi_3\}) = 1.6 & \end{array}$$

So M_R is a focussed significance function. Note also, M_R is discounting, additive, and capped.

Proposition 2. *Let M_R be a significance function. If M_R is focussed and capped, then M_R is discounting.*

Proof. Suppose M_R is not discounting. So there is no $\pi \in P_R$ such that $M(\{\pi\}) < 1$. So for all $\pi \in P_R$, $M(\{\pi\}) = 1$. Let $\pi_1, \pi_2 \in P_R$ be such that $\pi_1 \subset \pi_2$. So $M_R(\{\pi_1\}) = M_R(\{\pi_2\})$. But this contradicts the assumption that M_R is focussed. Therefore, M_R is discounting.

5 Using significance functions

We want to use a significance function to measure the degree of inconsistency arising in a given news report Φ when compared with some domain knowledge Δ. To do this we first identify the actual inconsistencies that arise between Φ and Δ and then apply the significance function.

Definition 17. *Let Φ be a news report and Δ be domain knowledge. The set of* **actual inconsistencies** *in Φ is*

$$D(\Phi, \Delta) = \{\{\phi_1, \ldots, \phi_n\} \subseteq \Phi \mid \Delta \vdash \neg(\phi_1 \wedge \ldots \wedge \phi_n)\}$$

So $D(\Phi, \Delta)$ contains the subsets of Φ that are rebutted by Δ.

The following two properties just stress that the actual inconsistencies depend on the amount of information in both the news report and the domain knowledge.

Proposition 3. *If Φ is a news report and Δ is domain knowledge, then $D(\emptyset, \Delta) = \emptyset$ and $D(\Phi, \emptyset) = \emptyset$.*

Proposition 4. *Let Φ and Φ' be news reports and Δ and Δ' be sets of domain knowledge.*

$$\text{If } \Phi \subseteq \Phi' \text{ then } D(\Phi, \Delta) \subseteq D(\Phi', \Delta)$$

$$\text{If } \Delta \subseteq \Delta' \text{ then } D(\Phi, \Delta) \subseteq D(\Phi, \Delta')$$

We characterize a form of cumulatively in the actual inconsistencies using the following definition and proposition.

Definition 18. *For any set of literals α, the conjunction of the literals is denoted $\bigwedge \alpha$. So, if $\alpha = \{\alpha_1, .., \alpha_n\}$, then $\bigwedge \alpha = \alpha_1 \wedge .. \wedge \alpha_n$.*

Proposition 5. *If $\pi_1, \pi_2 \in D(\Phi, \Delta)$, then $\pi_1 \cup \pi_2 \in D(\Phi, \Delta)$. The converse does not hold.*

Proof. If $\pi_1 \in D(\Phi, \Delta)$, then $\Phi \vdash \bigwedge \pi_1$ and $\Delta \vdash \neg(\bigwedge \pi_1)$. Similarly, if $\pi_2 \in D(\Phi, \Delta)$, then $\Phi \vdash \bigwedge \pi_2$ and $\Delta \vdash \neg(\bigwedge \pi_2)$. So, $\Phi \vdash (\bigwedge \pi_1) \wedge (\bigwedge \pi_2)$, and $\Delta \vdash \neg((\bigwedge \pi_1) \wedge (\bigwedge \pi_2))$. This implies $\pi_1 \cup \pi_2 \in D(\Phi, \Delta)$. For the converse, consider $\Phi = \{\phi_1, \phi_2\}$ and $\Delta = \{\neg\phi_1 \vee \neg\phi_2\}$.

We can relate potential and actual inconsistencies as follows: Each set of actual inconsistencies is a multiple inconsistency. We formalize this as follows:

Proposition 6. *If R is a set of news reports, with $\Phi \in R$, and Δ is domain knowledge, then $D(\Phi, \Delta) \in Q_R$.*

Proof. By Definition 17, $\forall \pi \in D(\Phi, \Delta)$, $\pi \in \wp(\Phi) \backslash \emptyset$. Therefore, $\forall \pi \in D(\Phi, \Delta)$, $\pi \in P_R$. Hence, $D(\Phi, \Delta) \in Q_R$.

There is some redundancy inherent in the actual inconsistencies. We only require the minimal actual inconsistencies defined below.

Definition 19. *Let Φ be a news report and Δ be domain knowledge. The set of* **minimal actual inconsistencies** *in Φ is*

$$E(\Phi, \Delta) = \{\pi_i \in D(\Phi, \Delta) \mid \neg\exists\pi_j \in D(\Phi, \Delta) \text{ such that } \pi_j \subset \pi_i\}$$

Minimal actual inconsistencies are related to multiple inconsistencies in the following proposition.

Proposition 7. *If R is a set of news reports, with $\Phi \in R$, and Δ is domain knowledge, then $E(\Phi, \Delta)$ is a coherent multiple inconsistency.*

Proof. First note that $\forall\lambda \in Q_R$, if $\lambda' \subseteq \lambda$, then $\lambda' \in Q_R$. Therefore, if $D(\Phi, \Delta) \in Q_R$, then $E(\Phi, \Delta) \in Q_R$. From Proposition 6, we have $D(\Phi, \Delta) \in Q_R$, and so we have $E(\Phi, \Delta) \in Q_R$. Finally, by Definition 19, $\neg\exists\pi_i, \pi_j \in E(\Phi, \Delta)$ such that $\pi_j \subset \pi_i$, and so $E(\Phi, \Delta)$ is coherent.

The degree of inconsistency in a news report is just the assignment given by the significance function for the multiple inconsistency in the news report.

Definition 20. *Let Φ be a news report, Δ be domain knowledge, and M_R be a significance function. The* **degree of inconsistency** *in Φ is given by $M_R(E(\Phi, \Delta))$.*

Example 7. Let M_R be an additive significance function for Q_R. Let $\Phi = \{\phi_1, \phi_2, \phi_3\}$ be a news report in R, where $M_R(\{\{\phi_1\}\}) = M_R(\{\{\phi_2\}\}) = M_R(\{\{\phi_3\}\}) = 0.4$, and let $\Delta = \{\neg\phi_1, \neg\phi_2\}$ be domain knowledge. So $E(\Phi, \Delta) = \{\{\phi_1\}, \{\phi_2\}\}$, and $M_R(E(\Phi, \Delta)) = 0.8$. This means we regard the inconsistencies as not being sufficient grounds for invoking some action on the report.

Example 8. Let M_R be a classical significance function for Q_R. Let $\Phi = \{\phi_1, \phi_2, \phi_3\}$ be a news report in R, so $M_R(\{\{\phi_1\}\}) = M_R(\{\{\phi_2\}\}) = M_R(\{\{\phi_3\}\}) = 1$, and let $\Delta = \{\neg\phi_1, \neg\phi_2\}$ be domain knowledge. So $E(\Phi, \Delta) = \{\{\phi_1\}, \{\phi_2\}\}$, and $M_R(E(\Phi, \Delta)) = 2$. This means we regard each inconsistency as sufficient grounds for invoking some action on the report.

Example 9. Let M_R be a classical significance function for Q_R. Let $\Phi = \{\phi_1, \phi_2, \phi_3\}$ be a news report in R, so $M_R(\{\{\phi_1\}\}) = M_R(\{\{\phi_2\}\}) = M_R(\{\{\phi_3\}\}) = 1$, and let $\Delta = \{\neg\phi_1 \wedge \delta, \neg\phi_1 \wedge \gamma\}$ be domain knowledge. So $E(\Phi, \Delta) = \{\{\phi_1\}\}$, and $M_R(E(\Phi, \Delta)) = 1$. Note, however $MI(\Phi \cup \Delta) = \{\{\phi_1, \neg\phi_1 \wedge \delta\}, \{\phi_1, \neg\phi_1 \wedge \gamma\}\}$. So here we see $|E(\Phi, \Delta)| \neq |MI(\Phi \cup \Delta)|$.

We can now establish some relationships between significance functions and the MI function.

Proposition 8. *If Φ is a news report and Δ is domain knowledge, and $\Phi \cap \Delta = \emptyset$, then $|E(\Phi, \Delta)| \leq |MI(\Phi \cup \Delta)|$.*

Proof. If $|E(\Phi, \Delta)| = 0$, there is no minimal subset of Φ that is rebutted by Δ, and so $|MI(\Phi \cup \Delta)| = 0$. If $|E(\Phi, \Delta)| = 1$, let $E(\Phi, \Delta) = \{\pi\}$. So π is a minimal subset of Φ such that $\pi \cup \Delta \vdash \perp$. Hence there is a $\Gamma \in MI(\Phi \cup \Delta)$ such that $\pi \subset \Gamma$. Therefore, in this case $|E(\Phi, \Delta)| \leq |MI(\Phi \cup \Delta)|$. Now suppose $|E(\Phi, \Delta)| \geq 2$. Let $\pi_1, \pi_2 \in E(\Phi, \Delta)$. Since π_1 is a minimal subset of Φ such that $\pi_1 \cup \Delta \vdash \perp$, there is a $\Gamma_1 \in MI(\Phi, \Delta)$ such that $\pi_1 \subset \Gamma_1$. Similarly, there is a $\Gamma_2 \in MI(\Phi, \Delta)$ such that $\pi_2 \subset \Gamma_2$. Furthermore, there is no $\pi_i, \pi_j \in E(\Phi, \Delta)$ where $\pi_1 \subset \pi_2$ or $\pi_2 \subset \pi_1$. Hence, $\Gamma_1 \neq \Gamma_2$. To generalize, let $S(\pi) = \{\Gamma \in MI(\Phi, \Delta) \mid \pi \subset \Gamma\}$. For each $\pi \in E(\Phi, \Delta)$, $S(\pi)$ is non-empty, and for each $\pi_i, \pi_j \in E(\Phi, \Delta)$ $S(\pi_i) \cap S(\pi_j) = \emptyset$. Therefore, in these cases $|E(\Phi, \Delta)| \leq |MI(\Phi \cup \Delta)|$.

Proposition 9. *Let $\Phi \in R$ be a news report and let Δ be domain knowledge. If M_R is a capped additive significance function, and $\Phi \cap \Delta = \emptyset$, then $M_R(E(\Phi, \Delta)) \leq |MI(\Phi \cup \Delta)|$.*

Proof. If M_R is a capped additive significance function, then $\forall \lambda \in Q_R$, λ is coherent, implies $M(\lambda) \leq |\lambda|$. Therefore, by Proposition 8, $M_R(E(\Phi, \Delta)) \leq |MI(\Phi \cup \Delta)|$.

Proposition 10. *Let $\Phi \in R$ be a news report and let Δ be domain knowledge. If M_R is a classical significance function, and $\Phi \cap \Delta = \emptyset$, then $M_R(E(\Phi, \Delta)) \leq |MI(\Phi \cup \Delta)|$. However, in general, $M_R(E(\Phi, \Delta))$ is not equal to $|MI(\Phi \cup \Delta)|$.*

Proof. Since M_R is a classical significance function, we have M_R is a capped additive significance function. Therefore, by Proposition 9, $M_R(E(\Phi, \Delta)) \leq |MI(\Phi \cup \Delta)|$. As an example of $M_R(E(\Phi, \Delta))$ not being equal to $|MI(\Phi \cup \Delta)|$, consider $\Delta = \{\neg\phi \wedge \psi_1, \neg\phi \wedge \psi_2\}$ and $\Phi = \{\phi\}$. So $E(\Phi, \Delta) = \{\{\phi\}\}$, but $MI(\Phi \cup \Delta) = \{\{\phi, \neg\phi \wedge \psi_1\}, \{\phi, \neg\phi \wedge \psi_2\}\}$. So $M_R(E(\Phi, \Delta)) = 1$, but $|MI(\Phi \cup \Delta)| = 2$.

If we have two or more news reports in R, then we can compare them using the significance function. The significance function induces a linear ordering over a set of reports.

Definition 21. *Let $\Phi_i, \Phi_j \in R$, and Δ is domain knowledge. If $M_R(E(\Phi_i, \Delta)) \leq M_R(E(\Phi_j, \Delta))$ then Φ_j is* **more inconsistent** *than Φ_i (according to M_R and Δ).*

The "more inconsistent than" ordering constitutes a comparison based on an aggregation of the number and significance of the minimal potential inconsistencies of each report that are rebutted by the domain knowledge.

6 Predicate ranking

We can consider the question of generating significance functions for particular applications. Here, we assume the set of news reports R is too large for a user to want or indeed be able to define a significance function. To address this, we introduce the notion of ranking of predicates to induce an ordering over potential inconsistencies.

Definition 22. *Let T be a set of predicate symbols. Then a* **predicate ranking** *is a pre-order relation $\leq_T$ over T. So we say β is* **more significant** *than α in α iff $\alpha \leq_T \beta$ holds. For notational convenience, $\alpha <_T \beta$ iff $\alpha \leq_T \beta$ and $\beta \not\leq_T \alpha$*

Example 10. Let $T = \{\texttt{temperature}, \texttt{humidity}, \texttt{precipitation}\}$. Then a predicate ranking is

$$\texttt{humidity} \leq_T \texttt{temperature} \qquad\qquad \texttt{temperature} \leq_T \texttt{precipitation}$$

So this says that we regard inconsistencies involving literals with the `precipitation` predicate symbol as being the most significant, and inconsistencies involving literals with the `humidity` predicate symbol as being the least significant.

We now need to consider sets of predicates so that we can order potential inconsistencies. For this, we require the following definition.

Definition 23. *Let T be a set of predicate symbols with a predicate ranking $\leq_T$. The* **lexicographic predicate ranking** *for T is denoted $\preceq_T$ and is defined as follows: For $X, Y \in \wp(P)$, where the maximal element in X according to $\leq_T$ is x_1, and the maximal element in Y according to $\leq_T$ is y_1:*

$X \preceq_T Y$ iff
(1) $x_1 <_T y_1$ or
(2) $x_1 \not<_T y_1$ and $y_1 \not<_T x_1$ and $Y \subset X$ or
(3) $x_1 \not<_T y_1$ and $y_1 \not<_T x_1$ and $Y \not\subset X$ and $X \not\subset Y$ and $X \backslash \{x_1\} \preceq_T Y \backslash \{y_1\}$

Example 11. We continue Example 10. The lexicographic predicate ranking is:

$$\{\texttt{humidity}\} \preceq_T \{\texttt{temperature,humidity}\}$$
$$\{\texttt{temperature,humidity}\} \preceq_T \{\texttt{temperature}\}$$
$$\{\texttt{temperature}\} \preceq_T \{\texttt{humidity,temperature,precipitation}\}$$
$$\{\texttt{humidity,temperature,precipitation}\} \preceq_T \{\texttt{humidity,precipitation}\}$$
$$\{\texttt{humidity,precipitation}\} \preceq_T \{\texttt{temperature,precipitation}\}$$
$$\{\texttt{temperature,precipitation}\} \preceq_T \{\texttt{precipitation}\}$$

Before we use the predicate ranking, we require the following subsidiary function that just gives the predicate symbols used in each multiple inconsistency.

Definition 24. *Let Φ be a news report in R. The predicates function is defined as follows:*

For $X \subseteq \Phi$, $Predicates(X) = \{p_i \mid p_i(r_j, t_k) \in X\}$

For $\{X_1, .., X_n\} \subseteq \wp(\Phi)$,
$$Predicates(\{X_1, .., X_n\}) = Predicates(X_1) \cup .. \cup Predicates(X_n)\}$$

Example 12. Continuing Example 1,

$$Predicates(\{\texttt{date(r1, 01Jan2001)}, \texttt{city(r1, London)}, \texttt{precipitation(r1, rain)}\}) \\ = \{\texttt{date}, \texttt{city}, \texttt{precipitation}\}$$

The predicate ranking can be used to generate a lexicographic ordering over Q_R so that inconsistencies based on more significant predicate symbols are more highly ordered.

Definition 25. *Let R be a set of reports. Let $\preceq_T$ be a predicate ranking where T is the set of predicate symbols in R. The* **inconsistency ranking** *is defined as follows, where $\lambda_i, \lambda_j \in Q_R$*

$$M_R(\lambda_i) \leq M_R(\lambda_j) \text{ if } Predicates(\lambda_i) \preceq_T Predicates(\lambda_j)$$

The inconsistency ranking is a constraint on the significance function M_R, though it does not uniquely determine it.

Example 13. We continue Example 10. If we choose M_R so that M_R is additive and that for all `r` and `t` the following hold,

$$M_R(\{\{\texttt{precipitation(r,t)}\}\}) = 1$$
$$M_R(\{\{\texttt{temperature(r,t)}, \texttt{precipitation(r,t)}\}\}) = 1$$
$$M_R(\{\{\texttt{humdity(r,t)}, \texttt{precipitation(r,t)}\}\}) = 1$$
$$M_R(\{\{\texttt{humdity(r,t)}, \texttt{temperature(r,t)}, \texttt{precipitation(r,t)}\}\}) = 1$$
$$M_R(\{\{\texttt{temperature(r,t)}\}\}) = 0.5$$
$$M_R(\{\{\texttt{temperature(r,t)}, \texttt{humdity(r,t)}\}\}) = 0.5$$
$$M_R(\{\{\texttt{humdity(r,t)}\}\}) = 0.5$$

then we have a discounting significance function that is consistent with the predicate ranking. Though, the significance function is not focussed.

Since the lexicographic predicate ordering does not lead to a focussed significance function, a linearization of $(T, \subseteq)$ that is consistent with the predicate ranking $\leq_T$ would be a useful alternative.

7 Discussion

It is common to spot inconsistencies between a news report and domain knowledge. Some of these inconsistencies may be more significant than others, and some news reports may contain more inconsistencies than others. Having a measure of the "degree of inconsistency" of a news report can help in deciding how to act on it. Though, we assume each tool based on significance functions is developed for a particular domain and for a particular user group.

Currently, we are extending the framework to take into account the text entries (the terms) used in the binary predicates in the news reports. For example, an inconsistency about the price of a new Ferrari would be more significant if the entry was 1000 Euros as opposed to 50,000 Euros. We are also investigating a generalization of Definition 3 that will allow nesting as used in XML or structured text [Hun00].

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