

MATH0011 Mathematical Methods 2

<i>Year:</i>	2024–2025
<i>Code:</i>	MATH0011
<i>Level:</i>	4 (UG)
<i>Normal student group(s):</i>	UG: Year 1 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	2
<i>Assessment:</i>	The final weighted mark for the module is given by: 20% programming component. 5% assessed homework (vector calculus). 75% unseen exam, including one question on the programming part. In order to pass the module you must have at least 40% in both the examination and the final weighted mark.
<i>Normal Pre-requisites:</i>	MATH0010
<i>Lecturer:</i>	Dr M Towers and Prof R Halburd

Course Description and Objectives

This module consists of two parts: first an introduction to programming for the mathematical sciences (4 weeks) and then an introduction to multivariable calculus (5 weeks).

Programming is becoming an increasingly important tool for mathematicians both in industry and in research. In view of this, some elements of basic scientific computation should be part of any modern undergraduate curriculum in Mathematics. Multivariable calculus on the other hand is of fundamental importance in a variety of fields of pure and applied mathematics such as electromagnetism, fluid mechanics, differential geometry and integration theory.

The aim of the first part is to introduce the students to the ideas of computer programming and its uses in scientific computing for science and its applications. The programming language Python will be considered in the course, but the underlying principles are general. Students should learn how to write accurate programs for the computational solution of mathematical problems.

The aim of the second part is to introduce the students to the ideas of the calculus of several variables, and to develop their understanding of functions of several variables, their derivatives and integrals.

Recommended Texts

The Python Tutorial <https://docs.python.org/3/tutorial/> takes you through the functionality of Python with lots of examples. The best way to learn Python is to practise: *Project Euler* <https://projecteuler.net/> is an online collection of over 600 mathematical programming problems that make excellent practice. Finally, Langtangen's book *A Primer on Scientific Programming with Python* <https://hplgit.github.io/primer.html/doc/pub/half/book.pdf> gives an introduction to using Python with a focus on using Python for mathematical problems; however, please note that many of the problems featured in this book are outside the scope of this course.

For the vector calculus part, there are many excellent texts available. In particular, *Calculus* (Gilbert Strang), of which chapters 13–15 are relevant to this course, is free online at <https://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/> You may also like the Schaum's Outline Series *Advanced Calculus*; *Vector Calculus* (Paul Matthews) or *Advanced Engineering Mathematics* (Erwin Kreyszig).

Detailed Syllabus

Programming:

- Introduction to Python, running code, input and output.
- Variables, arithmetic operations.
- Loops and conditionals.
- Types, strings, integers, floats.
- Lists, tuples, and dictionaries.
- Functions, local and global variables.
- Classes, instances and objects.
- Creating and importing modules.
- Debugging and error handling.
- Plotting.
- Matrices and vectors.
- Numerical differentiation and integration.

Multivariable calculus

- Coordinate systems. Polar coordinates in 2D. Cylindrical and spherical polar coordinates in 3D.
- Scalar functions of several variables: review of partial derivatives, linear approximations, the gradient, directional derivatives, tangent planes. Chain rule. Taylor series. Critical points of functions of two independent variables.
- Vector functions. Curves and velocity. Integral with respect to arc length.
- Double integrals. Triple integrals. Jacobian. Change of variables in multiple integrals.
- Vector fields. Line integrals. Divergence, curl. Scalar potential. Independence of path. Green's theorem. Surface integrals.