Suppose we wish to maximise

$$\prod_{i=1}^{T} p_i$$

subject to the conditions $T>0$, $p_i \geq 0$, $\sum_{i=1}^{T} p_i = C$ and $C \geq 0$.

Since logarithm is a monotonically increasing function for the range of $P$ of interest we can maximise (1) by maximising its logarithm. We do this by differentiating w.r.t. $p_i$ for $1<i<T$ subject to the constraints. The maximum value of (1) occurs when each of the partial derivatives is zero.

$$\log \left( \prod_{i=1}^{T} p_i \right) = \log \left( C - \sum_{i=2}^{T} p_i \right) + \sum_{i=2}^{T} \log p_i$$

$$\frac{\partial \log \prod_{i=1}^{T} p_i}{\partial p_i} = -\frac{1}{\left( C - \sum_{i=2}^{T} p_i \right)} + \frac{1}{p_i}$$

Setting each partial derivative to zero (and noting $p_i \neq 0$) yields

$$-\frac{1}{\left( C - \sum_{i=2}^{T} p_i \right)} + \frac{1}{p_i} = 0$$
\[-\left(C - \sum_{i=2}^{T} P_i\right) + P_1 = 0\]

\[-P_1 + P_i = 0\]

\[P_i = P_1\]

That is the $P_i$ when (1) is maximal are all equal. Since they still have to sum to C every $P_i = 1/C$.

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