Maximum Product of Non-Negative Numbers [1]

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Suppose we wish to maximise

$$\prod_{i=1}^{T} P_i \tag{1}$$

subject to the conditions $T>0$, $P_i \geq 0$, $\sum_{i=1}^{T} P_i = C$ and $C \geq 0$.

Since logarithm is a monotonically increasing function for the range of P of interest we can maximise (1) by maximising its logarithm. We do this by differentiating w.r.t. P_i for $1<i<T$ subject to the constraints. The maximum value of (1) occurs when each of the partial derivatives is zero.

$$\log\left(\prod_{i=1}^{T} P_i\right) = \log\left(C - \sum_{i=2}^{T} P_i\right) + \sum_{i=2}^{T} \log P_i$$

$$\frac{\partial \log \prod_{i=1}^{T} P_i}{\partial P_i} = -\frac{1}{\left(C - \sum_{i=2}^{T} P_i\right)} + \frac{1}{P_i}$$

Setting each partial derivative to zero (and noting $P_i \neq 0$) yields

$$-\frac{1}{\left(C - \sum_{i=2}^{T} P_i\right)} + \frac{1}{P_i} = 0$$

$$
\begin{aligned}
-\left(C - \sum_{i=2}^{T} P_i\right) + P_i &= 0 \\
-P_1 + P_i &= 0 \\
P_i &= P_1
\end{aligned}
$$

That is the P_i when (1) is maximal are all equal. Since they still have to sum to C every P_i =1/C.

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