Quantitative and Computational Finance

Code: GM21
Term: 2
Year: MSc
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Aims & Motivation

Mathematics provides a universal framework for innovation. Whilst some feel the interaction between mathematics and industry is far from optimal, the financial markets provides the best example of a successful partnership.

'Quantitative Finance' as a branch of modern finance is one of the fastest growing areas within the corporate world. Together with the sophistication and complexity of modern financial products, this exciting discipline continues to act as the motivating factor for new mathematical models and the subsequent development of associated computational schemes.

Alternative names for this subject area are *Mathematical Finance, Financial Mathematics* or *Financial Engineering*. Although relatively young, financial mathematics has developed rapidly into a substantial body of knowledge and established part of mathematical science.

Financial Engineering has the attraction of being one of only a few areas of mathematics that plays a central role in current developments in its domain of application. It has a reciprocal - relationship with the `real world' while it both draws from and has direct implications upon every-day financial practice in the commercial arena. Many practitioners agree that a responsible approach to even understanding the concepts requires the use of mathematics.

This is a course in the applied aspects of mathematical finance, in particular derivative pricing. The overall theme of the course is to develop the Partial Differential Equation (PDE) approach to the pricing of options. As well as a two hour examination during the summer term, students will undertake a short computing project where they will use numerical and computational techniques to perform derivative pricing.

Learning Outcomes

Students upon completion will obtain a basic flavour of the mathematical models and computational schemes used in investment banks and hedge funds, by quantitative analysts working in derivative pricing roles.
In addition to understanding the mathematics of stochastic differential equations (SDEs), students will be able to simulate these for underlying assets and derive the PDEs for pricing options on various asset classes.

During the computational project students will demonstrate their understanding of the numerical and computational methods, by formulating and solving a real life derivative pricing problem. The course will also be useful as a starting point and/or for generating interest for possible PhD research.

Content:

Simulation Methods in Finance: Brief introduction to Stochastic Differential Equations (SDEs) – drift, diffusion, Itô's Lemma. The statistics of random number generation in Excel. Simulating asset price SDEs in Excel.

Black-Scholes Framework: Similarity reduction and fundamental solution for the heat equation. Black-Scholes PDE: simple European calls and puts; put-call parity. The PDE for pricing commodity and currency options. Discontinuous payoffs – Binary and Digital options. The greeks: theta, delta, gamma, vega & rho and their role in hedging.

Computational Finance: Solving the pricing PDEs numerically using Explicit, Implicit and Crank-Nicholson Finite Difference Schemes. Stability criteria. Monte Carlo Technique for derivative pricing.

Fixed-Income World: Introduction to the properties and features of fixed income products; yield, duration & convexity. Stochastic interest rate models: stochastic differential equation for the spot interest rate; bond pricing PDE; popular models for the spot rate (Vasicek, CIR and Hull & White); solutions of the bond pricing equation; calibration.

Method of Instruction:

Mainly formal lectures with weekly problem sheets. A few computing classes (where appropriate) where students will be able to perform computer simulations and pricing.

Assessment:

The course has the following assessment components:

- Two hour written examination in the summer (90%)
- Individual Project (10%)

To pass this course, students must:

- Obtain an overall combined mark of at least 50%

Note:

Due to the mathematical nature of this module - a background in calculus, probability and differential equations obtained in an undergraduate degree (mathematics, science, engineering or economics) is a compulsory requirement for this course.

Resources:

Paul Wilmott Introduces Quantitative Finance – Paul Wilmott. J. Wiley & Sons. 2nd Edition, 2007