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ERRATA TO "RELATION ALGEBRA REDUCTS OF CYLNDRIC ALGEBRAS AND COMPLETE REPRESENTATIONS"

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Thanks to Tarek Sayed Ahmed for bringing an important error in [1] to my attention. In the abstract of [1], the fourth displayed equation claims wrongly that "$\exists$ has a winning strategy in $H(\mathsf{At}(\mathcal{A}))$ implies $\mathcal{A} \in \mathfrak{Ra}(\mathbf{CA}_\omega)$". Unfortunately, this turns out to be false. This line should be replaced by the weaker claim: "$\exists$ has a winning strategy in $H(\mathsf{At}(\mathcal{A}))$ implies there is $\mathcal{C} \in \mathbf{RCA}_\omega$ such that $\mathsf{At}(\mathfrak{Ra}\mathcal{C}) \cong \mathsf{At}(\mathcal{A})$." This weaker claim is already proved in [1, theorem 39].

Two lines down, in the final displayed equation of the abstract, the line "$\mathfrak{Ra}\mathbf{RCA}_\gamma \subseteq K \subseteq S_c\mathfrak{Ra}\mathbf{CA}_5$" should be replaced by "$S_c\mathfrak{Ra}\mathbf{RCA}_\gamma \subseteq K \subseteq S_c\mathfrak{Ra}\mathbf{CA}_5$." Whether $\mathfrak{Ra}\mathbf{RCA}_\omega$ is elementary or not is open.

These changes in the abstract require slight changes to theorem 39 and definition 40 and a more substantial change to theorem 45. Theorem 39 should be slightly strengthened as follows. "Let $\gamma \geq 5$, let α be a countable relation algebra atom structure. If $\exists$ has a winning strategy in $H(\alpha)$ then there is $\mathcal{C} \in \mathbf{RCA}_\gamma$ such that $\mathfrak{Ra}(\mathcal{C})$ is atomic and $\mathsf{At}\mathfrak{Ra}(\mathcal{C}) \cong \alpha$. The proof already shows that $\mathcal{C} \in \mathbf{RCA}_\omega$ and we may extend the result to ordinals $\gamma > \omega$ by redefining U_α to be $\{f \in {}^\gamma\mathsf{nodes}(N_a) : \{i < \gamma : f(i) \neq g(i)\}$ is finite$\}\}$.

In [1, definition 40], the final line "Let $\mathcal{A}$ be the complex algebra over α (so the domain consists of arbitrary sets of atoms)." should be replaced by "Let $\mathcal{A}$ be the *term algebra* of α — the countable subalgebra of the complex algebra of α, generated by α."

Theorem 45 is wrong. The correct statement should be "Let $\gamma \geq \omega$ and let K be any class of relation algebras such that $S_c\mathfrak{Ra}\mathbf{CA}_\gamma \subseteq K \subseteq S_c\mathfrak{Ra}\mathbf{CA}_5$. Then K is not closed under elementary subalgebras hence K is not an elementary class." The proof of this revised theorem can be simplified and completed without the use of the hypernetwork game H. Here, however, we aim to minimise the size of this errata. Accordingly, the corrected proof to theorem 45 is:

"Let $\mathcal{A}$ be the rainbow algebra of definition 40 and let $\mathcal{A}' \succeq \mathcal{A}$ be the countable elementary extension given by lemma 44. Since $\exists$ has a winning strategy in $H(\mathcal{A}')$, by theorem 39 there is $\mathcal{C} \in \mathbf{RCA}_\gamma$ such that $\mathsf{At}\mathfrak{Ra}(\mathcal{C}) \cong \mathsf{At}(\mathcal{A}')$. Let $\mathcal{C}' \supseteq \mathcal{C}$ be the McNeille completion of $\mathcal{C}$, this is a complete cylindric algebra and $\mathsf{At}\mathfrak{Ra}(\mathcal{C}') = \mathsf{At}\mathfrak{Ra}(\mathcal{C})$. Then $\mathcal{A}' \subseteq_c \mathfrak{Cm}(\mathsf{At}(\mathcal{A}')) = \mathfrak{Ra}(\mathcal{C}')$, by lemma 15, so $\mathcal{A}' \in S_c\mathbf{RCA}_\gamma$. But $\mathcal{A} \notin K$, by lemma 41."

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REFERENCES

[1] R Hirsch, *Relation Algebra Reducts of Cylindric Algebras and Complete Representations*, ***Journal of Symbolic Logic***, vol. 72(2) 2007, pp. 673–703.