UCL DEPARTMENT OF
COMPUTER SCIENCE

Research Note
RN/11/05

Money Management

20 January 2011

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Abstract

For a speculative investor, there are two aspects to optimizing a trading strategy. The first and most important goal of a trader is to achieve a positive expected risk-adjusted return. Once this has been achieved, the trader needs to know what percentage of his capital to risk on each trade. Within the trading fraternity this task is often know as *money management*. The underlying principals of money management apply to both gambling and trading, and were originally developed for the former. The problem of how to maximize growth of wealth has been solved: maximize the expected value of the logarithm of wealth after each period. However, most investors are unwilling to endure the volatility of wealth that such a strategy entails, and as John Maynard Keynes reminded us, in the long run, we're all dead. For this reason, a compromise between the optimal growth strategy and the security of holding cash is often recommended. This note reviews the literature on money management.

In a paper on the measurement of risk that launched ‘expected utility theory’, Bernoulli (1738) proposed that people have a logarithmic utility function. He noted that, as a consequence of this, when profits are reinvested, in order to measure the value of risky propositions one should calculate the geometric mean. The paper was later translated into English (Bernoulli, 1954). In an article on ‘Speculation and the carryover’ that focuses on cotton trading, Williams (1936) states that a speculator should bet on a representative future price, and points out that if his profits and losses are reinvested the method of calculating such a price is to choose the geometric mean of all the possible prices. In 1944 the mathematician John von Neumann and economist Oskar Morgenstern wrote *Theory of Games and Economic Behavior* (von Neumann and Morgenstern, 1944). Now a classic book, this is the work upon which modern-day game theory is based. In 1948 Claud Shannon published an article entitled ‘A mathematical theory of communication’ in two parts (Shannon, 1948). The paper established the discipline of *information theory* and became a classic. In short, he developed the concepts of information entropy and redundancy. Shannon asserted that binary digits could be transmitted over a noisy channel with an arbitrarily small probability of error if the binary digits were suitably encoded.

In 1956 a scientist working for Bell Labs, John Larry Kelly, Jr., brought together game theory and information theory when he published ‘A new interpretation of information rate’[1] (Kelly, 1956). He showed that in order to achieve maximum growth of wealth, at every bet a gambler should maximize the expected value of the *logarithm* of his capital, because it is the logarithm which is additive in repeated bets and to which the law of large numbers applies. The assumptions are that the gambler’s capital is infinitely divisible and all profits are reinvested. Money management systems which maximize the expected value of the capital are said to employ the *Kelly criterion*. Bellman and Kalaba (1957) considered the role of dynamic programming in statistical communication theory and generalized and extended Kelly (1956)’s results. As the first to introduce the Kelly criterion in an economic context, Latané (1959) showed that investors should maximize the geometric mean of their portfolios.

Breiman (1961) proved that using the Kelly criterion is asymptotically optimal under two criteria: (1) minimal expected time to achieve a fixed level of resources and (2) maximal rate of increase of wealth. It is only in continuous time that the results are *exact*. In 1962, Edward O. Thorp, an American maths professor, author and blackjack player wrote *Beat the Dealer* (Thorp, 1962), which became a classic and was the first book to prove mathematically that the house advantage in blackjack could be overcome by card counting. Thorp and Walden (1966) developed a winning strategy for a side bet in Nevada Baccarat and used the Kelly criterion to determine bet sizes. Thorp (1969) concluded that the Kelly criterion should replace the Markowitz criterion (Markowitz, 1959) as the guide to portfolio selection.

Hakansson (1970) considered the optimal investment and consumption strategies under risk for a class of utility functions and also gave the necessary and sufficient conditions for long-run capital growth. Radner (1971) was the first to employ a balanced investment strategy in the context of stochastic generalizations of the von Neumann model of economic growth. Thorp (1971) applied the Kelly criterion to portfolio choice. Samuelson (1971) showed that it is *not* the case that the geometric-mean strategy is optimal for any finite number of periods, however long, or that it becomes asymptotically a good approximation. Goldman (1974) showed that the policy of maximizing the expected logarithm of terminal wealth as applied to bounded utilities when the time horizon is long is not always optimal. Merton and Samuelson (1974) exposed the fallacy of the log-normal approximation to optimal portfolio decision-making over many periods. Miller (1975) showed that when the horizon is infinite, the investment policy of maximizing the expected log each period is optimal when the utility function depends only on the tail of the sequence representing the capital at each period. Kraus and Litzenberger (1975) developed a competitive equilibrium model of a market embodying heterogeneous beliefs. They assumed that each investor maximizes the expected logarithmic utility of his future wealth by selecting state-by-state claims to future wealth constrained only by his initial wealth. Friedman (1976) presented ‘Understanding and applying the Kelly criterion’ to the Third Conference on Gambling and Risk Taking in Las Vegas, Nevada in December 1976. Markowitz (1976) argued that in the sequence-of-games formalization of the maximum-expected-log rule the criterion for asymptotic optimality adopted by Merton and Samuelson (1974) and Goldman (1974) is unacceptable, because it violates the notion that only the normalized form of the game is necessary for comparing strategies. Economists take a dim view of the Kelly system, in a notable attack Samuelson (1979) wrote a journal article using words of only one syllable, ‘Why we should not make mean log of wealth big though years to act are long’ which was published in the normally polysyllabic *Journal of Banking & Finance*. His point was that ‘[w]hen you lose – and you *sure can* lose – with N large, you can lose real big.’

Bell and Cover (1980) used a game-theoretic model of a market by implementing a one-shot, two-player, constant-sum game where the goal of each investor is to maximize the probability of outperforming the opponent. They found that a Kelly investor has at least a 50% chance of out-returning any other gambler after just one trial. Thorp (1980) published ‘The Kelly money management system’ in the *Gambling Times* which detailed the Kelly formula. Finkelstein and Whitley (1981) extended the results of Kelly and Breiman and showed that a Kelly investor is never

[1] The original title was ‘Information theory and gambling’, but Kelly changed it to appease his employer.

behind any other gambler on average after any fixed number of bets. Wong (1981) showed that when using optimal proportional betting in blackjack, your expected win divided by your bet size is half of your expected arithmetic win rate. Gehm (1983) wrote *Commodity Market Money Management*, which includes Thorp's implementation of the Kelly criterion (Thorp, 1980). Ethier and Tavaré (1983) showed that the ratio of your return on investment under optimal proportional betting to your return on investment under constant betting converges to an exponential distribution with mean $\frac{1}{2}$ as your advantage tends to 0. Griffin (1984) considered different measures of win rate for optimal proportional betting. In a book about betting on horse racing, Ziemba and Hausch (1985) showed that for lengthy sequences of bets a Kelly investor's expected capital growth significantly exceeds that from alternative investment strategies. Pestien and Sudderth (1985) demonstrated how to control a diffusion to a goal in continuous-time. Heath et al. (1987) showed in continuous time that if one is managing a portfolio of stocks, bonds and cash and wishes to minimize the expected time to reach a given total worth, then the Kelly strategy is optimal. Ziemba (1987) simulated 1,000 seasons of betting on 700 horse races and demonstrated that proportional betting using the Kelly formula was superior to any other staking strategy. Algoet and Cover (1988) proved that maximizing conditionally expected log return given currently available information at each stage is asymptotically optimal, with no restrictions on the distribution of the market process. Ethier (1988) presented 'The proportional bettor's fortune' and showed that in terms of security, an optimal policy is often to keep all wealth in a riskless asset.

Ralph Vince wrote *Portfolio Management Formulas* (Vince, 1990) in which he popularized and extended Kelly's formula under the guise of a position sizing method he termed *optimal* f. Thomas Cover and Joy Thomas published *Elements of Information Theory* which includes chapters on 'Gambling and data compression' and 'Information theory and the stock market' (Cover and Thomas, 1991). Vince (1992) wrote *The Mathematics of Money Management*, in which he weds his 'optimal f' to the optimal portfolio. MacLean et al. (1992) considered how an investor should make the trade-off between maximal growth (i.e. using the Kelly criterion) and maximal security (e.g. minimizing drawdowns). Balsara (1992) authored the book *Money Management Strategies for Futures Traders*. Rotando and Thorp (1992) applied the Kelly strategy to long-term investment in S&P 500 stocks, and demonstrated some of the benefits and liabilities of that strategy. Aucamp (1993) provided both theory and evidence to indicate that the 'long-run' required for the geometric mean strategy to be optimal can be quite long in risky situations, but can 'virtually dominate' in a moderate number of plays in cases when risk is low. Vince (1995) published his third book on money management, which elaborates on his 'optimal f' and describes a new model for portfolio construction. In an excellent book chapter, Hakansson and Ziemba (1995) reviewed the theory of capital growth, in particular the growth-optimal investment strategy (the Kelly criterion). Browne and Whitt (1996) considered the *Bayesian* version of gambling and investment problems, where the underlying stochastic process has parameter values that are unobserved random variables, and derived a generalization of the Kelly criterion. Thorp (1997) presented a paper that discusses the use of the Kelly criterion in blackjack, sports betting and the stock market. Cetinkaya and Parlar (1997) provided a critique of the simple logarithmic assumption for the utility of terminal wealth and solved the problem with a more general utility function. They showed that in the general case the optimal policy is *not* myopic. Karatzas and Shreve (1998) published *Methods of Mathematical Finance* which includes a section on the maximization of the growth rate of wealth.

Aurell et al. (2000) used Kelly's theory to price derivatives in incomplete markets. Aurell and Muratore-Ginanneschi (2000) studied long-term growth-optimal strategies in a simple market with transaction costs. On an infinite time horizon the investment-optimal strategy consists of allowing the amount of capital invested in stocks to fluctuate freely within an interval around the value of the optimal investment in the absence of trading costs. Browne (2000) analysed some of the short-run properties of the Kelly strategy. Haigh (2000) interpreted the Kelly strategy in the context of spread betting. Evstigneev and Schenk-Hoppé (2002) prove that *any* self-financing constant proportions investment strategy yields a strictly positive exponential rate of growth of investor's wealth in a financial market in which prices are described by stationary stochastic processes and the price ratios are non-degenerate (provided that the investor trades in at least two stocks). In the first part of an easy-to-read two-part article Ziemba (2002) introduces Kelly betting. In the second part Ziemba (2003a) considers the application of the Kelly criterion to lotteries. Ziemba (2003b) gives an easy-to-read review of the Kelly criterion. Anderson and Faff (2004) traded a simple and publicly available trading rule in five futures markets and reinvested any profits via the 'optimal f' technique described by Vince (1990). They concluded that money management in speculative futures trading plays a more important role in trading rule profitability than previously considered by providing dramatic differences in profitability depending on how aggressively the trader capitalizes each futures contract. Ethier (2004) showed that the Kelly system maximizes the median of the gambler's fortune. Poundstone (2005) wrote *Fortune's Formula* which is a fascinating study of the connections between gambling, information theory, stock investing and applied mathematics, with Kelly's system central to the book. Leibfarth (2006) wrote a dumbed down item on money management in a popular magazine for traders. Piotrowski and Schroeder (2007) explain the Kelly criterion in terms of thermodynamics. Fabozzi et al. (2007) published *Robust Portfolio Optimization and Management*, a guide to portfolio construction using 'robust optimization'. Vince (2007) wrote *The Handbook of Portfolio Mathematics*, new material includes his implementation of drawdown as a risk metric. In their book *Scenarios for Risk Management and Global Investment Strategies*, Ziemba and Ziemba (2007) discuss scenarios for risk management and developing global investment strategies. McDonnell (2008) published *Optimal Portfolio Modeling*, an introduction to portfolio modelling using Excel and R. He combines the logarithmic utility function (Bernoulli, 1738) with the use of logarithms for maximum growth of wealth (Kelly, 1956), resulting in an iterated log function, $\log(1 + \log(1 + r))$, where r is return. Michaud and Michaud (2008) demonstrate the limitations of Markowitz mean-variance optimization, and use Monte Carlo resampling to address information uncertainty. Osorio (2009) employed the Kelly criterion to find the optimal leverage for investors with utility functions consistent with prospect theory (Kahneman and Tversky, 1979) and returns drawn from the Student's t-distribution (fat tails). In his fifth book on money management Vince (2009) describes *The Leverage Space Trading Model*, and presents a paradigm that seeks to maximize the probability of being profitable as opposed to maximizing profits. Lv and Meister (2009) studied the Kelly criterion in the continuous time framework. They showed that in a complete market there exists an optimal self-financing trading strategy that maximizes the logarithmic utility function.

MacLean et al. (2010) edited the book *The Kelly Capital Growth Investment Criterion: Theory and Practice*, which includes classic reprinted papers plus new material. Vince (2011) clarifies the distinction between the Kelly criterion and his own invention, 'optimal f'.

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