The Blagoveščenskiĭ Identity and the Inverse Scattering Problem

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The inverse scattering problem for the plasma wave equation

$$[\partial_t^2 - \triangle + q(x)]u(x,t) = 0$$

in three space dimensions is considered. Under certain assumptions about the potential, the time domain scattering problem can be formulated equivalently in the frequency domain. Time and frequency domain techniques are combined in the subsequent analysis.

The Blagoveščenskiĭ identity is generalised to the case of scattering data, assuming an inverse polynomial decay of the potential. This identity makes it possible to calculate the inner product of certain solutions of the plasma wave equation at a given time, if the corresponding incident waves and the scattering amplitude are known. In the case of a compactly supported potential, these inner products can be calculated for the time derivatives of all solutions.

If the potential is compactly supported, a variant of the boundary control method shows that using appropriate superpositions of plane waves as incident waves, it is possible to excite a wave basis over a compact set. Letting this set shrink to a point, the Blagoveščenskiĭ identity provides pointwise information about the solutions. When substituted into the plasma wave equation, this yields a method for solving the inverse problem.