Radiative Transfer Problem on Non-Convex Manifolds

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Abstract

We consider the radiative transfer problem on geodesically non-convex manifolds. The interest for non-convex regions rises for example in the optical tomography of human cortex, where the measurements must be made behind the non-scattering CSF-layer, whose inner boundary is geometrically all but regular.

The forward problem of radiative transfer is to find a solution $u : SM \to \mathbb{R}$ on the unit sphere bundle SM for the radiative transfer equation

$$Hu + \mu_a u = \mu_s \int_{S_x M} \theta(\xi, \xi') u(\xi') d\xi' \qquad , \qquad u|_{\partial_- SM} = f,$$

where H is the vector field on SM generating the geodesic flow, $\theta(\xi, \xi')$ is the scattering amplitude and f describes the sources at the inward boundary

$$\partial_- SM = \{ \, \xi \in S_x M \mid x \in \partial M \ , \ g(\nu, \xi) < 0 \, \}.$$

In the convex case the solutions of the radiative transfer problem are smooth, given any smooth boundary data and material parameters, but in the non-convex case this property ceases to hold due to the shadows cast by intruding parts of the boundary. However, much of the smoothness is still present in the solutions in the non-convex case, and here we aim at a sensible description for this sense of smoothness.