Asymptotic field formulations, MUSIC-type retrievals of small 3-D bounded dielectric and/or magnetic inclusions, and their application to dipole source and receiver arrays[1]

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Abstract. The identification of a collection of small 3-D bounded homogeneous inclusions, with arbitrary permittivity, conductivity and permeability, buried within a homogeneous or stratified medium is considered via time-harmonic electromagnetic means.

The applications envisaged are in environment, civil and military engineering, non-destructive testing of man-made structures, medical imaging, etc. Specific attention at the present stage is given to configurations possibly met in the identification of small objects within Earth subsoils.

The proposed approach uses Music-type algorithms, and enables us to achieve fast numbering, accurate localization, and in the best case (so-called well-separated inclusions) estimates of the electromagnetic and geometric parameters of the inclusions.

Those are of unknown finite number (say, m) and they are assumed to be, as already indicated, of small volume (typically, ellipsoidally shaped) with respect to the wavelength at the operation frequency (or to the skin depth in case of lossy media and diffusive phenomena), in addition to be located far enough from one another with respect to their characteristic size ε, and far enough from the interfaces if any.

The collection is illuminated by an array (e.g., linear, planar, yet this can be more complicated) of electric and/or magnetic dipoles with given orientation(s) (one will speak also of polarisations) located at some, not necessarily large distance from it (both near and far-field situations are encompassed), the case of aspect-limited data being in particular of good interest. (Plane-wave illuminations would be easily dealt with as well as more complex arrangements of sources and receivers whether useful.)

The resulting electric and/or magnetic field is collected by another array, possibly but not necessarily the same as the emitting one. This yields the $n_S d_S$ x $n_R d_R$ multistatic data (response) matrix that is characteristic of the collection for a given set of n_S dipolar sources with d_S orthogonal orientations and for a given set of n_R dipolar receivers with d_R orthogonal orientations, the least value of $n_S d_S$ and $n_R d_R$ being assumed to be properly (this aspect will be discussed later) larger than the number of inclusions. (Let us remind that the said matrix times its conjugate-transpose is but the well-known time-reversal matrix.)

The exposé itself will have two parts.

In the first part, it will be summarized how, starting from application of Green theorems to the Maxwell PDE satisfied by the electromagnetic fields, and as an extension to the case of unbounded media of previous theoretical analyses (e.g., H. Ammari and H. Kang, *Reconstruction of Small Inclusions from Boundary Measurements*, Springer (2004)), one is able to derive rigorous asymptotic formulations of the scattered electric and/or magnetic fields as series expansions in terms of integral powers of the average size ε, here limiting ourselves for simplicity to the leading-order term (power 3). Such formulations involve the classical (static) electric and magnetic, 3 x 3 polarization dyads (diagonal ones in their eigen coordinate system) associated to the inclusions —the Perfectly Electrically Conducting (PEC)

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case follows from the introduction of an infinite conductivity and a null permeability. In short, these formulations account for the propagation of the wavefield from the sources (with given polarisations) to the inclusions and (upon scattering) from the inclusions to the receivers (the collected signals will evidently depend upon their polarisations) via the electric and magnetic dyads of the embedding medium (reduced to transverse ones in far-field situations).

In the second part, now as a rather involved extension of previous works led in 2-D scalar scattering situations, e.g., H. Ammari, E. Iakovleva and D. Lesselier, "A MUSIC algorithm for locating small inclusions buried in a half-space from the scattering amplitude at a fixed frequency," *Multiscale Model. Simul.* 3, 597–628 (2005), it will be shown how the eigenvalue structure of the multi-static data matrix (the elements of which are equated to the asymptotically derived leading-order field components) can be employed within the framework of the MUSIC (standing for MUltiple SIgnal Classification) method in order to retrieve either dielectric inclusions (including the purely conductive case), or magnetic inclusions, or combinations thereof, with the limiting PEC case as well.

In practice this means carrying out two steps: (i) first, the calculation of singular values (and corresponding eigenvectors), the number of nonzero ones depending upon the number m and the electromagnetic nature of the inclusions and upon the sources and receivers' geometrical arrangement and polarization ($3m$ values at most in the dielectric or magnetic case, $6m$ at most in the general case, including PEC); (ii) second, the orthogonal projection of a properly built vector propagator (say, from the sources to every test point within a properly sampled 3-D search space) onto the null space (in the situation just mentioned, the right null space) of the multi-static matrix, coincidence with an inclusion being then associated to a peak of the inverse norm of the projection (yielding a spot of high amplitude within a 3-D image), the make-up of the indicator being itself function of the nature of the inclusions, if known (the purely dielectric case leads to the simplest form). In addition, scalar products of the singular vectors spanning the signal subspace by the aforementioned propagator can be displayed (here, this amounts to backpropagation from the receiver array onto the search space).

It should be emphasized that the above does produce singular values proportional to the volumes of the inclusions times their electric or magnetic contrast with respect to their immediate embedding in case of spherical inclusions (involving the polarization dyads for general ellipsoids, being remarked that, in view of the leading-order term of the fields, more complicated shapes reduce to ellipsoidal equivalents) for well-separated inclusions (refer to investigations by M. Fink, C. Prada, A. J. Devaney, and other authors).

Work is still on-going on this aspect of resolution, and in the present contribution, one will only pursue the theoretical analysis in the situation of a single ellipsoidal inclusion with arbitrary electromagnetic parameters and location vis-à-vis the source and receiver arrays (again, those being not necessarily the same ones). The singular vectors and values will in particular be exhibited in closed form. Let us notice that this enables us to retrieve as a particular case the results (dielectric or PEC sphere with symmetric location with respect to the source/receiver array) of D. H. Chambers and J. G. Berryman, "Analysis of the time-reversal operator for a small spherical scatterer in an electromagnetic field," *IEEE Trans. Antennas Propagat.* 52, 1729-1738 (2004).

Numerical examples will then be given, in order to illustrate at least in preliminary fashion (work again is not fully completed at the time of writing) the main features of the approach for both asymptotically exact (de facto committing the inverse crime) and severely noisy data calculated for electromagnetic and geometric configurations that might model the probing of a small number of inclusions buried in subsoils.

Finally, generalization of the above approach when two inclusions appear close enough to one another to allow electromagnetic coupling, will be briefly considered, via the introduction of proper polarization dyads combining the individual ones (this amounts to setting up and retrieving an equivalent ellipsoidal inclusion), time permitting.