Approximate Gauss-Newton methods for data assimilation

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Abstract

Variational data assimilation is a technique for retrieving the state of a system using observed data, through the minimization of a cost function constrained by a numerical model. The method is used in numerical weather prediction to obtain the initial conditions for a weather forecast. In practice this leads to a very large nonlinear optimization problem, of several million variables, which must be solved in real-time. In order to solve this problem efficiently, operational forecasting centres have implemented what is called an `incremental' version, whereby the solution to the full minimization problem is approximated by the minimization of a sequence of linear quadratic cost functions, each of which is constrained by the linearization of the full nonlinear numerical model. This procedure can be shown to be equivalent to applying a standard Gauss-Newton iteration method to minimize the original nonlinear problem.

In practice this procedure is too expensive to apply exactly in operational weather forecasting and various approximations must be made. Here we examine two types of approximation used commonly in data assimilation. Firstly, we examine `truncated' Gauss-Newton methods, where the inner linear problem is not solved exactly, and secondly we examine `perturbed' Gauss-Newton methods where the true linearized inner problem is approximated by a simplified linear problem. We establish conditions which ensure that these approximate methods converge. The results are illustrated with a numerical example.