Calderon's inverse conductivity problem and quasiconformal maps

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June 21, 2005

Abstract

In 1980 A. P. Calderon posed the following problem: Suppose $\Omega \subset R^n$ is smooth and bounded, $\sigma \in L^\infty(\Omega)$ is bounded away from zero, and that $u \in H^1(\Omega)$ is the weak solution of

$$div(\sigma \nabla u) = 0$$

$$u|_{\partial\Omega} = f$$

where $f \in H^{1/2}(\partial\Omega)$. Define the Dirichlet - to - Neumann map $\Lambda_\sigma : H^{1/2}(\partial\Omega) \to H^{-1/2}(\partial\phi)$ by

$$\Lambda_\sigma(f) = \sigma \frac{\partial u}{\partial \nu} \ |_{\partial\Omega}$$

where ν is the unit outer normal to $\partial\Omega$. Caldern's problem now reads: Does Λ_σ iniquely determine σ? In physical terms this can be refrased as: Can one determine the conductivity of a body by measuring voltages and currents on it's surface.

The inverse problem to determine σ from Λ_σ is also known as *Electrical Impedance Tomography*. It has been proposed as a valuable diagnostic tool especially for detecting breast cancer and for monitoring hart and lungs.

In these lecture we will describe a new theory that combines inverse boundary value problems for conductivity to quasiconformal maps and to non-linear Fourier transform in two dimensions. We will demonstrate how this theory leads to a complete constructive solution of Calderns inverse conductivity problem in the plane. To show this we introduce complex geometric optics solutions and work out Beals-Coifman theory for the conductivity equation. We also introduce a generalized Hilbert transform on the boundary of the unknown object and derive a uniquely solvable integral equation to recover the boundary values of the geometric optics solutions. The existence of these solutions is based on the theory of quasiconformal maps that are generalizations of conformal maps and were introduced by Groetsch in 1920's.

The work is a joint study with K. Astala from Helsinki.