Notions of Scattering Support

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Abstract

I will discuss discuss the inverse source and inverse scattering problems for the monochromatic (fixed wavenumber) Helmholtz equation. The basic problem is to describe a source, or scatterer, of a wave based on observations of the far field (a solution to the Helmholtz equation far away from the source or scatterer).

Neither the inverse source problem nor certain versions of the inverse scattering problem have unique solutions. That is, there can be many different sources that produce the same measured data. In order to compute some meaningful information, one must either assume the source has a special form (e.g. a sum of point sources, or the indicator function of convex set), or alternatively, identify something that all sources that produce that data must have in common.

I will take the second approach and describe some notions of scattering support. These are sets which support a source that can produce the measured data, and are the smallest possible within a restricted class of sets. The estricted classes I will describe are:

- convex sets
- well-separated unions of convex sets
- sets which are star-shaped about a fixed point.

I will describe several examples of sources that radiate the same far field to show that, without restrictions of the sort mentionned above, such a smallest source may not exist.