UNIVERSITY COLLEGE LONDON
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COMP0142: Problem Set 0

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1. Consider the following vectors and matrices:

$$\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 3 & -4 \\ -2 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -1 & -4 & 5 \\ -2 & 2 & 3 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \\ 2 & 2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

(a) Calculate the eigenvectors and eigenvalues of $\mathbf{A}$.

State whether these expressions make sense and evaluate them if they do:

(b) $\mathbf{v} \cdot \mathbf{w}$

(c) $\mathbf{A}^{-1}$

(d) $\mathbf{A}^T$

(e) $\mathbf{BC}$

(f) $\mathbf{CB}$

(g) $\mathbf{AC}$

(h) $\mathbf{A} + \mathbf{B}$

(i) $\mathbf{D} + \mathbf{A}$

(j) $\det\mathbf{A}$

(k) $\mathbf{v}^T\mathbf{A}\mathbf{w}$

(l) $\mathbf{D}^{-1}$

2. Consider the following function:

$$f(x,y) = \frac{2x + \ln y}{2x^2 - 3y + 1}$$

   (a) What is $\frac{\partial f}{\partial x}$?

   (b) What is $\frac{\partial f}{\partial y}$?

   (c) What is $\frac{\partial^2 f}{\partial x \partial y}$?

3. Consider the following function:

$$g(x) = \sum_{i=1}^{n} (x - y_i)^2$$

   where $\{y_i \in \mathbb{R}\}_{i=1}^{n}$ are all distinct.

   Which value of x achieves the minimal value of $g(x)$?

4. Demonstrate, using Taylor's expansion:

$$(1 - x) < e^{-x}$$

5. Consider the following probability density function (pdf):

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$.

(a) What is the name usually given to this pdf?

(b) State the mode of $p(x)$.

(c) Demonstrate that the stationary point of $p(x)$ is equal to the mode.

6. Suppose that X is a discrete random variable, with outcomes 0 and 1, with a distribution characterised such that $\Pr(X = 1) = \theta$.

(a) What is the mean of X?

(b) What is the variance of X?

(c) Assuming that a sequence of n samples are drawn iid from this distribution. State the log-likelihood function for this sequence.

(d) Now assume that we observe one such sequence of outcomes: $\{1, 1, 0, 0, 1\}$. Use the technique of maximum likelihood estimation to infer a value for θ.